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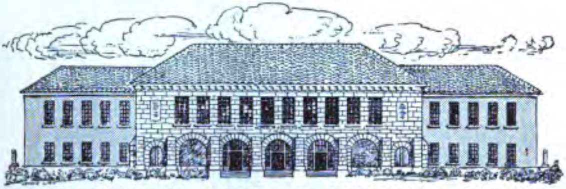

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EXAMPLES IN PHYSICS

EXAMPLES IN PHYSICS

BY

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PREFACE

THE value of the mental training obtained by solving algebraical problems and geometrical riders has so long been acknowledged that these form an essential part in all mathematical teaching. Although similar practice is quite as necessary in studying physical science, it is by no means equally easy for the student of physics to obtain it, for only the more recent text-books contain any numerical examples, and these are generally insufficient in number and not carefully graduated. It is quite common to find students who have a correct knowledge of the general principles of physics, and can apply it intelligently in making a physical measurement, but who are yet unable to solve an easy problem or to calculate the results of their experimental work.

There can be no doubt that the best way of acquiring the necessary practice is by means of a regular series of quantitative experiments in the laboratory carried on side by side with the more general work of the lecture-room; but such concurrent work is not always practicable, especially with large classes and in the earlier stages. Just as the student of dynamics has at first to confine his attention to questions of a more or less ideal nature, so in some departments of experimental physics (for example in electrostatics) the beginner must for a while content himself with somewhat theoretical problems in place of laboratory work.

The examples in the present book (amounting to over one thousand in number) consist for the most part of questions and problems framed for the use of the Junior and Middle Physics Classes at the Aberystwyth College. To these have been added (at the end of each chapter) questions from various College, University and Scholarship papers of recent years. A list of some of the more important examination papers from which these have been selected will be found on page 45, and the source from which each question is taken is in every case acknowledged. A considerable number of typical examples have been solved, and answers (with occasional hints for solution) will be found at the end of the book. Explanatory paragraphs have been inserted in the hope of giving assistance where experience seemed to show that it was most needed, but I have endeavoured not to trench upon the recognised province of the text-books.

The book has not been written with a view to the requirements of any special examination, but I have made use of portions of the MSS. in teaching classes of students taking the Intermediate Science and Preliminary Scientific Courses of the London University, and believe it will be found suitable for students who are preparing for these examinations.

For assistance in reading proofs and working out answers my best thanks are due to Mr. B. B. Skirrow, B.A., of the Mason Science College, Birmingham; to my assistant, Mr. R. W. Stewart; to Mr. F. W. Shurlock, B.A., of the Carmarthen Training College; and to one of my students, Mr. A. H. Leete.

D. E. JONES.

UNIVERSITY COLLEGE, ABERYSTYWYTH,
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ERRATA.

Page 25, Ex. 17, *for* velocity *read* acceleration.
Page 27, Ex. 40, *for* 56 lbs. *read* 49 lbs.
Page 111, Ex. 117 (last line), *for* 50° C. *read* – 50° C.
Page 112, Ex. 119 (last line), *for* volume *read* pressure.
Page 127, 6th line from bottom, *for* $ms\theta$ *read* mst.
Page 153, Ex. 5, *for* 4 yards *read* 4 feet.

(ANSWERS.)

Chap. I. (p. 247), **42**, *for* 18 feet *read* 16 feet. **53**, *for* 9·5 gm. *read* 14·58 gm.

Chap. II. (p. 250), **78**, sp. gr. of solid = 0·5; of liquid = 2. **66**, Resultant pressure = 62·5 lbs.

Chap. III. (p. 252), **48**, *for* 132°·3 *read* 131°·7.

Chap. IV. (p. 253), **48**, *for* 6·25 gm. *read* 625 gm.

Chap. V. (p. 254), **6**, *for* 10^{10} *read* 10^{9}. **29**, *for* 24,396 units *read* 24,124·4 units. **47**, *for* 2·85 *read* 2·87.

Chap. VI. (p. 255), **26**, *for* 1 ft. 1 in. *read* $\frac{12}{13}$ ft. **27**, *for* inverted *read* erect.

Chap. VII. (p. 256), **1**, 341 metres per sec. **7**, 32·4 cm.

EXAMPLES IN PHYSICS

INTRODUCTION

1. Units.—In order to measure any physical quantity, we have first to select as our *unit*, or standard of reference, a quantity of the same kind as that to be measured. The ratio between the quantity and the selected unit is called the *numerical value* or *measure* of the quantity. Suppose that we have to measure a definite length l, and that we adopt as our unit a length L: the numerical value (n) of the length to be measured will be

$$n=\frac{l}{\mathrm{L}},$$

where n may be any number, whole or fractional.

2. Fundamental Units.—All physical quantities can be expressed in terms of three fundamental units, the choice of which depends upon the ease and certainty with which the standard quantities so selected can be compared with other quantities of the same kind. We might choose as our fundamental quantities a definite length, a definite force, and a definite interval of time: other units, such as those of mass and work, could be deduced from these. But on account of the difficulty of devising a permanent standard of force, the value of which would not change from place to place, such a choice would not be advisable.

The fundamental units usually adopted are those of *length*, *mass*, and *time;* these three elements can be measured with great accuracy, and standards of length and mass can easily be copied, and compared with the original standards.

THE C.G.S. SYSTEM OF UNITS.

A committee of the British Association has recommended the adoption of the centimetre, the gramme, and the second as the three fundamental units. Other units derived from these are distinguished by the letters "C.G.S." prefixed, these being the initial letters of the three fundamental units.

3. Derived Units.

Velocity.—The C.G.S. unit of velocity is the velocity of a point which moves over one centimetre in a second.

Acceleration.—The C.G.S. unit of acceleration is that of a point whose velocity increases by one unit per second. The numerical value of the acceleration due to gravity (g) is 978·10 at the equator, 980·94 at Paris, 981·17 at Greenwich, and 983·11 at the pole.

Force.—The C.G.S. unit of force is that force which, acting upon a mass of one gramme for a second, generates in it a velocity of one centimetre per second.

Special names are given to some of these units; thus the C.G.S. unit of force is called the *dyne*. Assuming the value of g to be 981 (as we shall do throughout), we see that a dyne is $\frac{1}{981}$ of the weight of a gramme.

Work and Energy.—The C.G.S. unit of work is the work done by a dyne acting through a distance of one centimetre, and is called an *erg*. The same name is applied to the unit of *energy*, for energy is measured by the amount of work which it represents. Since the weight of a gramme is 981 dynes, the work done in raising one gramme vertically through one centimetre against the action of gravity is 981 ergs.

Practical Units and Index Notation.—In any uniform system some of the units must be inconveniently large, while others are so small that the quantities with which we have to deal are represented by very large numbers. Electricians find it convenient to use a system of "practical units," each of which bears to the corresponding C.G.S. unit a ratio which is some multiple or submultiple of 10. Thus the **volt** is equal to 100,000,000 C.G.S. units of potential; the **farad** is $\frac{1}{1,000,000,000}$ of the C.G.S. unit of capacity. The prefixes *mega-* and *micro-* are used to signify "one million" and "one-millionth part" respectively. Thus a *megadyne* is a force of one million dynes; a *microfarad* denotes a capacity of one-millionth of a farad.

When very large or very small numbers have to be expressed, it is convenient to adopt the index system of notation, in which numbers are expressed as the product of two factors, the second of which is a power of 10; and it is usual to choose the factors so that the first contains only one integral digit. Thus the velocity of light, which is 300,400 kilometres per second, is expressed as $3{\cdot}004 \times 10^{10}$ centimetres per second. A megadyne is 10^6 dynes; a farad is $10^{-9}\left(=\frac{1}{10^9}\right)$ and a microfarad is 10^{-15} C.G.S. unit of capacity.

Power, Activity, or Rate of doing Work.—The C.G.S. unit of power is the power of doing work at the rate of one erg per second. The corresponding practical unit, called the **Watt**, is the power of doing work at the rate of 10^7 ergs per second. A horse-power is equal to 746 watts.

Pressure.—The C.G.S. unit of intensity of pressure is a pressure of one dyne per square centimetre. It would be convenient if the pressure of a *megadyne per square centimetre* (10^6 C.G.S. units) were adopted as the normal atmospheric pressure: this standard would correspond to a barometric height of 75 centimetres, but, as compared with any barometric standard, would have the advantage of being independent of the value of g.

Heat.—The C.G.S. unit of heat is the amount of heat required to raise the temperature of a gramme of water through one degree centigrade. The dynamical equivalent of one heat-unit in ergs is $4{\cdot}2 \times 10^7$: this quantity is called the mechanical equivalent of heat or "Joule's equivalent," and is usually represented by the letter J.

4. C.G.S. Electrostatic Units.

Quantity.—The unit quantity of electricity is that quantity which, when placed (in air) at a distance of one centimetre from an equal and similar quantity, repels it with a force of one dyne.

Potential.—Unit difference of potential exists between two points when the work done against the electrical forces in moving unit quantity of electricity from the one point to the other is one erg.

Capacity.—A conductor is said to have unit capacity when a charge of one unit of electricity raises its potential from zero to unity.

Magnetic Units.

Strength of Pole.—A magnetic pole is said to have unit strength when it repels an equal and similar pole, placed at a distance of one centimetre from it, with a force of one dyne.

Strength of Field.—A magnetic field is said to have unit intensity (or strength) when a unit magnetic pole placed in it is acted upon by a force of one dyne.

Electro-magnetic Units.

Current.—The unit of current is that current which, when flowing along a wire one centimetre in length bent into the form of a circular arc of one centimetre radius, acts with a force of one dyne upon a unit pole placed at the centre of the circle.

Quantity.—The electro-magnetic unit of quantity is the quantity of electricity which in one second passes any section of a conductor in which unit current is flowing.

Electromotive Force or Difference of Potential.—Unit electromotive force[1] exists between two points when the work done against the electrical forces in moving unit quantity of electricity from the one point to the other is one erg.

Resistance.—A conductor is said to possess unit resistance when unit difference of potential between its ends causes unit current to flow through it.

Capacity.—A conductor has unit capacity when a charge of one unit of electricity raises its potential from zero to unity.

5. **Practical Units.**—The following system of units, based upon the C.G.S. electro-magnetic units, was devised by the British Association committee, and is in general use among practical electricians. It will be noticed that in this system the units of current, electromotive force, and resistance have been chosen so as to be of suitable magnitude for the electrical measurements which most frequently occur.

Current and Quantity.—The practical unit of current is the **ampère**, and is one-tenth (10^{-1}) of the C.G.S. electro-magnetic unit of current. It follows that the **coulomb**, or practical unit of quantity, is also one-tenth of the corresponding C.G.S. unit.

---

[1] Usually contracted into "E.M.F."

Electromotive Force.—The practical unit of E.M.F. is called the **volt**, and is 10^8 C.G.S. units. This is a little less than the electromotive force of a Daniell cell, the E.M.F. of a standard Daniell of the Post-Office pattern being 1·08 volt.

Capacity.—A conductor is said to have a capacity of one **farad** when it is charged to a potential of one volt by a coulomb of electricity. The farad is 10^{-9} of the C.G.S. electro-magnetic unit of capacity.

Resistance.—A conductor is said to possess a resistance of an **ohm** when a difference of potential of one volt between its ends causes a current of one ampère to flow through it. The ohm is therefore equal to 10^9 C.G.S. units of resistance.

Material standards, intended to represent the ohm as above defined, were issued by the B.A. committee, but their resistance is now found to be somewhat too small: and, for the sake of distinction, these standards and the copies of them which have since been made are known as "**B.A. Units.**" According to the best determinations of Lord Rayleigh and others

1 B.A. unit = 0·987 true ohm.

The B.A. unit has the same resistance as a column of mercury one square millimetre in cross-section and 104·8 centimetres long; whereas the true ohm would probably be represented by a column 106·2 centimetres long.

At the International Conference of Electricians held at Paris in 1884, it was agreed that the resistance of a column of mercury 106 centimetres long and 1 square millimetre in section, at the temperature of melting ice, should be adopted as the **legal ohm.**

6. Change of Units.—We have seen (§ 1) that the numerical value n of a length l is given in terms of the unit-length L by the equation.

$$n=\frac{l}{\mathsf{L}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Here we notice, in the first place, that the numerical value of a concrete quantity varies *directly* as the quantity itself, and *inversely* as the unit employed in measuring it. From equation (1) we have—

$$l=n\mathsf{L} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This second equation gives us a complete expression for the length l, an expression which consists of **two** parts—the first being a number (n), and the second a quantity (L) of the same kind as that under consideration, and which we call the unit. Our everyday expressions for all physical magnitudes are, in fact, phrases which consist of a numerical and a denominational part; thus we speak of a length of *ten yards*, and we say that *ten yards* are equal to *thirty feet.* This last statement involves a change of units,—a process which is perfectly easy when we have only to deal separately with units of length, mass, or time; but which becomes more difficult when two or more of the units have to be simultaneously changed. In dynamical problems which involve a change of units, it is usual to change the units one at a time; but this process becomes very laborious when the fundamental units are involved in a complex manner in those derived from them, as is the case with most electrical units. In proceeding with the general theory of units, we shall consider first, as a simple example, the principle involved in the statement that

10 yards = 30 feet.

Let l denote, as before, the length to be measured and n its numerical value when L is the unit of length. We wish to find its numerical value n' when the unit-length is L'. Now

$$n=\frac{l}{\mathsf{L}}, \text{ and } n'=\frac{l}{\mathsf{L}'},$$

$$\therefore\ l=n\mathsf{L}=n'\mathsf{L}',$$

or

$$n'=n\times\frac{\mathsf{L}}{\mathsf{L}'}.$$

The value of the quantity $\frac{\mathsf{L}}{\mathsf{L}'}$, which is the ratio of the first unit-length to the second, is called the *change-ratio* from the first system to the second. It is the factor by which the numerical value of the quantity in the first system must be multiplied in order to obtain its numerical value in the second system. In the case under consideration

$$\frac{\mathsf{L}}{\mathsf{L}'}=\frac{\text{yard}}{\text{foot}}=\frac{3 \text{ feet}}{1 \text{ foot}}=3,$$

and

$$\therefore\ n'=n\times\frac{\mathsf{L}}{\mathsf{L}'}=n\times 3=10\times 3=30.$$

DIMENSIONS OF PHYSICAL QUANTITIES.

7. Velocity.—We shall next consider how the measure of a velocity changes when the units by which it is measured are changed (using, as above, thick letters to represent units, and italics to represent the concrete quantities). Let V denote the unit of velocity based upon L and T as the units of length and time, V' the unit of velocity in a second system in which the units of length and time are L' and T'; and let v denote a concrete velocity such that a space l is described in the time t.

If n denote the measure (or numerical value) of this velocity in terms of the unit V, and n' its value in terms of the second unit V′, then

$$n=\frac{v}{\mathrm{V}},\ \text{and}\ n'=\frac{v}{\mathrm{V}'}\quad .\quad .\quad .\quad .\quad .\quad (1)$$

or

$$v=n\mathrm{V}=n'\mathrm{V}'.$$

Now the measure of a velocity is the number of units of space $\left(\frac{l}{\mathrm{L}}\right)$ described in unit time,

$$\left.\begin{aligned}\therefore n&=\frac{l}{\mathrm{L}}\div\frac{t}{\mathrm{T}},\ \text{or}\ n=\frac{l}{\mathrm{L}}\cdot\frac{\mathrm{T}}{t},\\ \text{and}\qquad n'&=\frac{l}{\mathrm{L}'}\div\frac{t}{\mathrm{T}'},\ \text{or}\ n'=\frac{l}{\mathrm{L}'}\cdot\frac{\mathrm{T}'}{t}\end{aligned}\right\}\quad .\quad .\quad (2)$$

From equations (1) and (2) we have

$$\mathrm{V}=v\cdot\frac{\mathrm{L}}{l}\cdot\frac{t}{\mathrm{T}},\ \text{and}\ \mathrm{V}'=v\cdot\frac{\mathrm{L}'}{l}\cdot\frac{t}{\mathrm{T}'}.$$

But

$$v=n\mathrm{V}=n'\mathrm{V}',$$

$$\therefore nv\cdot\frac{\mathrm{L}}{l}\cdot\frac{t}{\mathrm{T}}=n'v\cdot\frac{\mathrm{L}'}{l}\cdot\frac{t}{\mathrm{T}'},$$

which may be written in the form

$$n\cdot\frac{\mathrm{L}}{\mathrm{T}}=n'\frac{\mathrm{L}'}{\mathrm{T}'}\quad .\quad .\quad .\quad .\quad .\quad (3)$$

Equation (3) enables us to find the measure (n') of the velocity in the second system, when the relations between the fundamental units L and L′, T and T′ are known. Comparing it with the equation

$$n\mathrm{V}=n'\mathrm{V}'\quad .\quad .\quad .\quad .\quad .\quad (4)$$

we see that the unit of velocity varies *directly* as the unit of length, and *inversely* as the unit of time. This is usually expressed by saying that the *dimensions* of the unit of velocity are of one degree in length, and minus one degree in time; or that the dimensions of velocity are $\frac{\mathrm{L}}{\mathrm{T}}$ or LT^{-1}.

It should be noticed that $\frac{L}{T}$ is not a number, or a ratio in the strict Euclidean sense; it would, perhaps, be better to write it in the form L/T, the *solidus* or mark / standing for the word *per*. It is in this sense that the symbol $\frac{L}{T}$ is used: it indicates that in measuring a velocity we have to divide (not in the usual, but in a more extended sense) a length by a time. When, therefore, we write

$$V = \frac{L}{T},$$

we mean that the unit of velocity (V) is such that the space L is described per time T.

8. Acceleration.—Proceeding with this reasoning we shall find that in acceleration time is involved twice. For acceleration is measured by the increase of velocity per unit time; so that if A denote the unit of acceleration, A is equal to V per T, or $= V/T$, and since we have already seen that

$$V = \frac{L}{T},$$

it follows that

$$A = \frac{V}{T} = \frac{L}{T^2}.$$

We can arrive at the same result more formally as follows :—

Let A, V, L and T represent respectively the units of acceleration, velocity, length and time in one system, A', V', L' and T' the corresponding quantities in a second system; and let a denote an acceleration such that the velocity v is generated in the time t. If n be the measure of the quantity in the first system,

$$n = \frac{a}{A}.$$

But the measure of an acceleration is the number of units of velocity generated per unit of time, so that

$$n=\frac{v}{\mathsf{V}}\div\frac{t}{\mathsf{T}}=\frac{v}{\mathsf{V}}\cdot\frac{\mathsf{T}}{t},$$

$$\therefore\ \mathsf{A}=a\frac{\mathsf{V}}{v}\cdot\frac{t}{\mathsf{T}},$$

and similarly

$$\mathsf{A}'=a\frac{\mathsf{V}'}{v}\cdot\frac{t}{\mathsf{T}'}.$$

But since the quantity measured in both systems is the same, we have,

$$a=n\mathsf{A}=n'\mathsf{A}' \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$na\frac{\mathsf{V}}{v}\cdot\frac{t}{\mathsf{T}}=n'a\frac{\mathsf{V}'}{v}\cdot\frac{t}{\mathsf{T}'},$$

$$\therefore\ n\frac{\mathsf{V}}{\mathsf{T}}=n'\frac{\mathsf{V}'}{\mathsf{T}'},$$

or

$$n\cdot\frac{\mathsf{L}}{\mathsf{T}^2}=n'\frac{\mathsf{L}'}{\mathsf{T}'^2} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

A comparison of equations (5) and (6) shows that the unit of acceleration varies directly as the unit of length and inversely as the *square* of the unit of time; in other words, that its dimensions are $\frac{\mathsf{L}}{\mathsf{T}^2}$ or LT^{-2}.

The following example will illustrate the way in which these equations are applied:—

Ex. 1. *Express the acceleration due to gravity in terms of the mile and the hour as the units of length and time, its value being* 32 *when the foot is the unit of length, and the second the unit of time.*

From (6) we have directly

$$n'=n\cdot\frac{\mathsf{L}}{\mathsf{L}'}\cdot\frac{\mathsf{T}'^2}{\mathsf{T}^2},$$

an equation which gives us the required measure (n') in the new system, when the relations between the fundamental units in the old and new systems are known. In the example $n=32$,

$$\frac{L}{L'}=\frac{\text{foot}}{\text{mile}}=\frac{1\cdot\text{foot}}{(1760\times 3)\text{ feet}}=\frac{1}{5280},$$

$$\frac{T'}{T}=\frac{\text{hour}}{\text{second}}=\frac{3600\text{ seconds}}{1\text{ second}}=3600,$$

$$\therefore n'=32\times\frac{1}{5280}\times(3600)^2=78545\cdot\dot{4}\dot{5}.$$

9. Force, Work, and Power.—Before proceeding to give the dimensions of other derived units in mechanics, it may be well to point out the considerations which determine the choice of any new unit based upon the fundamental quantities or upon derived units which have already been fixed. We shall take the unit of force as our example.

According to the second law of motion, force is measured by the change of momentum which it produces, *i.e.*

$$f\propto(\text{rate of change of } mv),$$

$$\therefore f\propto ma,$$

(where a denotes acceleration), or

$$f=k\cdot ma.$$

The units of mass and acceleration are already fixed, but we may make the unit of force whatever we please, and it will obviously be most convenient to choose it so that the constant multiplier k shall be equal to unity. Our equation will now become

$$f=ma.$$

Now suppose m and a to be each equal to unity; then f will also be equal to unity. Thus our unit of force (F) is defined as being that force which produces unit acceleration in unit mass. We may therefore write

$$F=M\cdot A,$$

but

$$A=\frac{L}{T^2},$$

$$\therefore\ \mathrm{F}=\frac{\mathrm{ML}}{\mathrm{T}^2}=\mathrm{MLT}^{-2},$$

an equation which gives the dimensions of force.

Since work is measured by the product of force into the distance through which the force acts, the dimensions of work will be those of force multiplied by length, or

$$\mathrm{W}=\mathrm{MLT}^{-2}\times\mathrm{L}=\mathrm{ML}^2\mathrm{T}^{-2}.$$

The power (or activity) of an agent is measured by the rate at which it does work ; hence the dimensions of power are

$$\frac{\mathrm{ML}^2\mathrm{T}^{-2}}{\mathrm{T}}=\mathrm{ML}^2\mathrm{T}^{-3}.$$

Knowing the dimensions of these quantities, we can perform the change of units without going through the lengthy reasoning of §§ 7 and 8 ; we shall indicate the general method to be followed, but it will be best understood by reference to the actual examples given. [See also equations (5) and (6) in § 8.]

Let q be any concrete quantity, and let its measure be n in terms of the unit Q, which is based upon the fundamental units M, L, and T ; we wish to find its measure n' in a new system in terms of the unit Q′ which is based upon M′, L′, and T′. Since the quantity measured in both systems is the same,

$$q=n\mathrm{Q}=n'\mathrm{Q}' \quad . \quad . \quad . \quad . \quad (7)$$

Let the dimensions of Q be $\mathrm{M}^x\mathrm{L}^y\mathrm{T}^z$; substituting for Q and Q′ in equation (7) we have

$$q=n\cdot\mathrm{M}^x\mathrm{L}^y\mathrm{T}^z=n'\cdot\mathrm{M}'^x\mathrm{L}'^y\mathrm{T}'^z,$$

and
$$\therefore\ \frac{n'}{n}=\left(\frac{\mathrm{M}}{\mathrm{M}'}\right)^x\left(\frac{\mathrm{L}}{\mathrm{L}'}\right)^y\left(\frac{\mathrm{T}}{\mathrm{T}'}\right)^z,$$

or
$$n'=n\,.\left(\frac{\mathrm{M}}{\mathrm{M}'}\right)^x\left(\frac{\mathrm{L}}{\mathrm{L}'}\right)^y\left(\frac{\mathrm{T}}{\mathrm{T}'}\right)^z. \quad . \quad . \quad (8)$$

Ex. 2. *Find the number of dynes in a poundal (the poundal being the British absolute unit of force, based upon the pound, foot, and second).*

Referring to equations (7) and (8) we see that, since $n=1$, the required number n' is the change-ratio or multiplier for changing from British to C.G.S. units of force. The dimensions of force are MLT^{-2}, so that $x=1$, $y=1$, and $z=-2$. T and T', the units of time, are the same (one second) in both systems.

$$\frac{\mathsf{M}}{\mathsf{M}'}=\frac{\text{pound}}{\text{gramme}}=453{\cdot}6,$$
$$\frac{\mathsf{L}}{\mathsf{L}'}=\frac{\text{foot}}{\text{centimetre}}=30{\cdot}48,$$
$$\therefore\ n'=453{\cdot}6\times 30{\cdot}48=13825{\cdot}8.$$

Ex. 3. *Find the value of a horse-power in watts, a horse-power being equivalent to* 550 *foot-pounds per second, and the value of g being* 32·18.

As the foot-pound is a gravitation unit, we shall first have to reduce to the corresponding absolute unit by multiplying by g—

$$550\text{ foot-pounds}=550\times 32{\cdot}18\text{ foot-poundals.}$$

The dimensional equation for finding the equivalent rate of working in C.G.S. units (ergs per second) is,

$$550\times 32{\cdot}18\times\mathsf{ML^2T^{-3}}=n'\times\mathsf{M'L'^2T'^{-3}}.$$

The units of time (T and T') are the same in both systems; and, as in Example 2,

$$\frac{\mathsf{M}}{\mathsf{M}'}=453{\cdot}6,\text{ and }\frac{\mathsf{L}}{\mathsf{L}'}=30{\cdot}48,$$
$$\therefore\ n'=550\times 32{\cdot}18\times 453{\cdot}6\times(30{\cdot}48)^2$$
$$=745{\cdot}8\times 10^7.$$

Thus 1 horse-power $=745{\cdot}8\times 10^7$ ergs per second, or (since 1 watt $=10^7$ ergs per second)

$$=745{\cdot}8\text{ watts.}$$

10. Magnetic and Electrical Units. — The dimensions of the most important of these are given below, and it will be useful practice for the student to deduce them from the corresponding physical laws, as we have done in the preceding articles.

DIMENSIONS OF MAGNETIC UNITS.

Strength of magnetic pole . . .	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$
Magnetic moment of magnet . .	$M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$
Strength of magnetic field . . .	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$

Ex. 4. *The dimensions of magnetic intensity (or strength of field) are* $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$, *and the horizontal intensity of the earth's magnetic force at Aberystwith is* 0·1774 *in C.G.S. units: what is its value in British (foot-grain-second) units?*

The intensity is the same, whatever units we employ to measure it. Let x be its numerical value in the British system, in which the unit of field intensity is H', the corresponding unit in the C.G.S. system being H; then

$$0{\cdot}1774\,H = xH',$$

or

$$0{\cdot}1774\,M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} = xM'^{\frac{1}{2}}L'^{-\frac{1}{2}}T'^{-1},$$

and

$$\therefore x = 0{\cdot}1774\left(\frac{M}{M'}\right)^{\frac{1}{2}}\left(\frac{L}{L'}\right)^{-\frac{1}{2}}\left(\frac{T}{T'}\right)^{-1}.$$

Now since 1 gramme = 15·43 grains, and 1 foot = 30·48 centimetres,

$$\frac{M}{M'} = \frac{\text{gramme}}{\text{grain}} = 15{\cdot}43, \text{ and } \frac{L}{L'} = \frac{\text{centimetre}}{\text{foot}} = \frac{1}{30{\cdot}48},$$

and the units of time (T and T') are the same in both systems.

Thus

$$x = 0{\cdot}1774 \times (15{\cdot}43)^{\frac{1}{2}} \times \left(\frac{1}{30{\cdot}48}\right)^{-\frac{1}{2}},$$

$$= 0{\cdot}1774 \times \sqrt{15{\cdot}43 \times 30{\cdot}48} = 3{\cdot}847.$$

Ex. 5. *Assuming Coulomb's law (the law of inverse squares), to find the dimensions of the unit of quantity in the electrostatic system.*

According to the law of inverse squares, the force exerted between two bodies charged with quantities q and q' of electricity, and situated at a distance d from one another, is proportional to the product of the charges and inversely proportional to the square of the distance. Choosing our unit of quantity in accordance with the definition of § 4, we may write this in the form

$$f = \frac{qq'}{d^2}.$$

If we suppose that $q' = q$, we have $q^2 = d^2 f$, or $q = d\sqrt{f}$, so that the dimensions of the unit of quantity are $\mathsf{L} \times \sqrt{\mathsf{MLT}^{-2}}$ or $\mathsf{M}^{\frac{1}{2}}\mathsf{L}^{\frac{3}{2}}\mathsf{T}^{-1}$.

DIMENSIONS OF ELECTROSTATIC UNITS.

Quantity of electricity . . .	$\mathsf{M}^{\frac{1}{2}}\mathsf{L}^{\frac{3}{2}}\mathsf{T}^{-1}$
Electrostatic potential . . .	$\mathsf{M}^{\frac{1}{2}}\mathsf{L}^{\frac{1}{2}}\mathsf{T}^{-1}$
Capacity	L
Strength of current . . .	$\mathsf{M}^{\frac{1}{2}}\mathsf{L}^{\frac{3}{2}}\mathsf{T}^{-2}$
Resistance	$\mathsf{L}^{-1}\mathsf{T}$

DIMENSIONS OF ELECTRO-MAGNETIC UNITS.

Strength of current . . .	$\mathsf{M}^{\frac{1}{2}}\mathsf{L}^{\frac{1}{2}}\mathsf{T}^{-1}$
Quantity of electricity . . .	$\mathsf{M}^{\frac{1}{2}}\mathsf{L}^{\frac{1}{2}}$
Potential or E.M.F. . . .	$\mathsf{M}^{\frac{1}{2}}\mathsf{L}^{\frac{3}{2}}\mathsf{T}^{-2}$
Resistance	$\mathsf{L}\,\mathsf{T}^{-1}$
Capacity	$\mathsf{L}^{-1}\mathsf{T}^{2}$

11. Approximate Calculations. — Arithmetical working may often be abbreviated by devices such as the contracted methods of multiplication and division of decimals. The degree of approximation to which the calculation must be carried out depends upon the accuracy of the data given. Physical measurements are never absolutely correct. If, then, we have to calculate out the results of an experiment made by a method which is liable to an error of (say) one in a thousand, it would be labour thrown away to carry out the calculation to more than four or five significant figures. Now it frequently happens, in working out the results of physical experiments, that the "uncorrected result" has to be multiplied by one or more correcting factors (each nearly equal to unity) in order to obtain the "corrected result;" and it is to the manipulation of these factors that the student's attention is now directed.

Suppose that the experiment under consideration consists in measuring the distance between two points by means of a steel metre scale, the length of which at 0° C. is known to be 1·00057 metre; and suppose further that the measurement is carried out at a temperature of 15° C. The steel scale expands on heating, and its length at 15° is greater (Chap. III.) than its length at 0° in the ratio of 1·00018 to 1. If the uncorrected distance, as determined by direct measurement, is d, then the true distance (corrected for error of scale *and* error through temperature) will be

$$d' = d(1+\alpha)(1+\beta),$$

where $1+\alpha = 1{\cdot}00057$, and $1+\beta = 1{\cdot}00018$.

Now $(1+\alpha)(1+\beta) = 1+\alpha+\beta+\alpha\beta$; and since $\alpha = 0{\cdot}00057$ and $\beta = 0{\cdot}00018$, $\alpha\beta = 0{\cdot}000,000,1026$, so that the error caused by neglecting this last term would only be 1 in 10,000,000. The measurement itself would probably not be correct to 1 in 1,000,000, so that we may safely adopt the approximation, and write

$$d' = d(1+\alpha+\beta) = d \times 1{\cdot}00075. \quad (q.p.)$$

Again, suppose that in reducing our observations we have to multiply the uncorrected result by $\frac{1+\alpha}{1+\beta}$. (This correcting factor occurs in reducing observed barometric heights to 0° C., and other examples of its use are given in Chap. III.) By ordinary algebraical division we have

$$\frac{1+\alpha}{1+\beta} = 1+\alpha-\beta+\left(\frac{\alpha\beta+\beta^2}{1+\beta}\right).$$

We have already seen that if both α and β are small quantities compared with unity, their product may be neglected; and the same is true for α^2 and β^2.

Thus $$\frac{1+\alpha}{1+\beta} = 1+\alpha-\beta. \quad (q.p.)$$

Ex. 6. *Find correctly to three decimal places the value of*

$$15{\cdot}24 \times \frac{1{\cdot}00217}{1{\cdot}00192}.$$

$$\begin{aligned} \frac{1{\cdot}00217}{1{\cdot}00192} &= 1 + 0{\cdot}00217 - 0{\cdot}00192, \quad (q.p.) \\ &= 1{\cdot}00025, \text{ and therefore} \end{aligned}$$

$$\begin{aligned} 15{\cdot}24 \times \frac{1{\cdot}00217}{1{\cdot}00192} &= 15{\cdot}24 \times 1{\cdot}00025, \\ &= 15{\cdot}24 + (15{\cdot}24 \times 0{\cdot}00025), \\ &= 15{\cdot}24 + 0{\cdot}000381 = 15{\cdot}24381. \end{aligned}$$

The answer, correctly to three decimal places, is 15·244.

The student can easily verify for himself the following results, which are approximately correct when the quantities α and β are small compared with unity (so that their squares and higher powers may be neglected).

Table of Approximate Relations.

$$(1+\alpha)(1+\beta)=1+\alpha+\beta$$

$$(1+\alpha)(1-\beta)=1+\alpha-\beta$$

$$(1+\alpha)^2=1+2\alpha \qquad (1-\alpha)^2=1-2\alpha$$

$$(1+\alpha)^3=1+3\alpha \qquad (1-\alpha)^3=1-3\alpha$$

$$\sqrt{1+\alpha}=1+\tfrac{1}{2}\alpha \qquad \sqrt{1-\alpha}=1-\tfrac{1}{2}\alpha$$

$$\frac{1}{1+\alpha}=1-\alpha \qquad \frac{1}{1-\alpha}=1+\alpha$$

$$\frac{1+\alpha}{1+\beta}=1+\alpha-\beta.$$

12. Use of Logarithms.—It is proved in treatises on algebra that different powers of any fixed number can be multiplied by adding together the indices of those powers. We may assume that a list can be drawn up, giving the indices of the powers of some fixed number which are equal to every whole number, say from 1 to 10,000. Such a list is called a table of logarithms, and the fixed number is called the *base* of the system of logarithms: we may therefore define the logarithm of a number to a given base as being the index of that power of the base which is equal to the given number. Thus if $a^x=n$, then x is called the logarithm of n to the base a.

From motives of convenience the number 10 is chosen as the base of the common system of logarithms, and a table will be found at the end of this book giving the decimal parts (to four places) of the common logarithms of numbers from 1 to 9999.

To find the Logarithm of a Number from the Table.—The first two figures of the number are to be found in the left-hand column, and the third in the first series of figures (0 to 9) in the top column; the number opposite the first two figures, and below the third, is the decimal part of the logarithm. When the number whose logarithm is required contains four figures, the fourth figure is to be looked for in the second series of figures (1 to 9) in the top column; the proportional part, which is found opposite the first two figures and below the fourth, is to be added to the part of the logarithm already found, the right-hand figure of the proportional part being added to the right-hand figure of the logarithm.

The integral part of a logarithm is called the *characteristic*; the decimal part is called the *mantissa.*

The characteristic of the logarithm of a number may be determined by inspection. For

$$10^0 = 1,\ 10^1 = 10,\ 10^2 = 100, \text{ etc.},$$

and it therefore follows that the logarithm of any number between 1 and 10 is a positive decimal fraction: the logarithm of any number between 10 and 100 lies between 1 and 2, and the logarithm of any number between 100 and 1000 lies between 2 and 3. Hence the rule:—

(1.) *The characteristic of the logarithm of a number greater than unity is one less than the number of integral figures in that number.* Thus

$$\begin{aligned}\log\ 3{\cdot}14 &= 0{\cdot}4969\\ \log 31{\cdot}4 &= 1{\cdot}4969\\ \log 314 &= 2{\cdot}4969\\ \log 3140 &= 3{\cdot}4969.\end{aligned}$$

Again, since $10^0 = 1$, and $10^{-1} = 0{\cdot}1$, it follows that the logarithm of any number between 0 and 0·1 is a negative decimal fraction, and may therefore be written in the form—

$$-1 + \text{a decimal fraction.}$$

Similarly the logarithm of any number between 0·1 and 0·01 (*i.e.* between 10^{-1} and 10^{-2}) may be written in the form—

$$-2 + \text{a decimal fraction,}$$

the decimal part being always kept *positive.* Hence the rule:—

(2.) *The characteristic of the logarithm of a number less than unity is negative, and is one more than the number of ciphers immediately following the decimal point.*

Thus the logarithm of 0·314 is $-1 + 0{\cdot}4969$, which is abbreviated thus: $\bar{1}{\cdot}4969$; the logarithm of 0·0314 is $\bar{2}{\cdot}4969$, and so on.

The operation of multiplication is performed by adding together the logarithms of the numbers which are to be multiplied: the sum is the logarithm of their product. Division is performed by subtracting the logarithm of the divisor from that of the dividend: the remainder is the logarithm of the quotient. The manner in which these operations are carried out, and the method of finding a number when its logarithm is given, will be best explained by an example.

Ex. 7. *To find the value of* $\frac{453\cdot6\times30\cdot48}{(2\cdot54)^2\times13600}$ *by the use of four-place logarithms.*

$$\begin{array}{ll} \log 453\cdot6 & = 2\cdot6567 \\ \log \ \ 30\cdot48 & = 1\cdot4840 \\ \cline{2-2} \log \text{dividend} & = 4\cdot1407 \\ \log \text{divisor} & = 4\cdot9431 \\ \cline{2-2} \log \text{quotient} & \underline{\underline{= \bar{1}\cdot1976}} \end{array}$$

$$\begin{array}{ll} \log (2\cdot54)^2 & \\ \quad = 2 \log 2\cdot54 & \\ \quad = 2\times0\cdot4048 & = 0\cdot8096 \\ \log 13600 & = 4\cdot1335 \\ \cline{2-2} \quad \log \text{divisor} & = 4\cdot9431 \end{array}$$

0·1976 is not one of the logs given in the table: the next lower one is 0·1959, which is the log of 1·57. Now 0·1976 − 0·1959 = 0·0017. Looking along the row (in which the log is given) for 17, we find that it stands in a column headed by the figure 6, and this is the fourth figure of the number. Lastly, by rule (2), we see that $\bar{1}\cdot1976 = \log 0\cdot1576$. The value of the fraction is therefore 0·1576.

A full account of the methods of logarithmic calculation will be found in Chambers's *Mathematical Tables*, and these may be used for more accurate work; but the table of four-place logarithms at the end of this book will be found sufficient for working out most of the problems given.

The student is advised to practise the methods of approximation indicated in § 11, and to make himself thoroughly familiar with the use of logarithms, as a large amount of arithmetical calculation will thus be avoided.

In working out examples he should aim at something more than merely getting a correct numerical answer: diagrams or rough sketches should be given wherever they render the solution more intelligible, and formulæ should not be quoted without explanation unless the relations which they express are perfectly well known and easily remembered. In particular, every step in the reasoning should be carefully thought out and clearly explained, for the solution of problems is not so much an end itself as a means of acquiring a thorough and intimate acquaintance with physical laws.

CHAPTER I

DYNAMICS

Note.—In all the examples, excepting where otherwise stated, the numerical value of g is taken as 981 when the centimetre and second are the units of length and time, and as 32 when the foot is the unit of length.

The abbreviation *cm.* is used for *centimetre(s)*.
" *gm.* " *gramme(s)*.
" *c.c.* " *cubic centimetre(s)*.

In examples on change of units, the following (approximate) relations may be assumed—

1 foot = 30·48 cm.
1 inch = 2·54 cm.
1 pound = 453·6 gm.

1. State and discuss Newton's First Law of Motion, and show that it provides us with a definition of force.

2. Enunciate Newton's Second Law; state the exact meaning of the phrase "change of motion" as used by him, and explain how the law enables us to measure forces.

3. Starting from Newton's Second Law of Motion, show how to obtain a definition of the C.G.S. unit of force (the dyne). If the weight of a gramme be taken as the unit of force, what is the unit of mass?

4. A force of 25 units acts upon a mass 10: find the acceleration produced, and the space described in 30 seconds from rest.

5. A force of 100 dynes acts upon a mass of 25 grammes for 5 seconds: what velocity does it generate?

6. A constant force acting upon a mass of 30 grammes causes it to move through 10 metres in 3 seconds, starting from rest: what is the value of the force in dynes?

7. A force of 1,000,000 dynes acts upon a body for 10 seconds, and gives it a velocity of a metre per second: find the mass of the body in grammes.

8. How long must a force of 5 units act upon a body in order to give it a momentum of 3000 units? (The unit of momentum is that of a gramme moving at the rate of one centimetre per second.)

9. During what time must a constant force of 60 dynes act upon a kilogramme in order to generate in it a velocity of 3 metres per second?

10. What force acting upon a mass of 50 grammes for one minute will produce a velocity of 45 centimetres per second?

11. A body moving with uniform velocity in a circle is commonly said to be acted on by "centrifugal force." Discuss the correctness of this expression, stating whether the quantity referred to is really a force. Is its action centrifugal?

12. State and explain Newton's Third Law of Motion, and give examples of its application. If the earth attracts the moon with a force F, what is the attraction exerted by the moon upon the earth?

13. The mass of a gun is 2 tons, and that of the shot is 14 lbs. The shot leaves the gun with a velocity of 800 feet per second: what is the initial velocity of the recoil?

14. A 56-lb. shot is projected with velocity v from a gun, the mass of which (together with its carriage) is 6 tons. Express, in terms of v, the velocity of recoil of the gun.

15. Do you consider *weight* to be an essential property of matter? State clearly what distinction you would draw between *mass* and *weight;* and illustrate your remarks by reference to the force required (1) to open a large iron gate, well balanced and swinging upon good hinges, and (2) to lift up the same gate when lying on the ground.

16. Explain what is meant by "the acceleration due to gravity." If its numerical value be 32 when the unit of length is the foot and the unit of time the second, what is its value when the unit of length is the yard and the unit of time the minute? (See § 8, Ex. 1.)

17. A 4-oz. weight is suspended from a spring-balance which is carried in a balloon; what will be its apparent weight as shown by the index (1) when the balloon is ascending with an uniform acceleration of 8 feet per second, (2) when it is descending with a velocity of 16 feet per second?

18. What do you understand by the phrase "weight of a pound"? The British unit of force (called a poundal) is defined as being that force which, acting upon a pound mass for one second, generates in it a velocity of one foot per second: how many poundals are there in a pound weight?

19. Explain the distinction between gravitation measure and the absolute measure of force. Show also how the one may be expressed in terms of the other, finding, for example, the number of dynes in a gramme weight.

20. Express the weight of 10 kilogrammes in dynes, and the value of a dyne in terms of a gramme weight.

21. Calculate the value of a pound weight in dynes. (See note on p. 23.)

22. A force of 980 dynes acts vertically upwards upon a body of mass 5 grammes, at a place where $g = 981$: find the acceleration of the body.

23. A force equal to a weight of 10 lbs. acts upon a mass of 25 lbs.: what is the acceleration produced, and what momentum will be generated in 5 minutes?

24. A body of mass 6 lbs. is acted upon by a force of 30 poundals: find its velocity and momentum at the end of half a minute.

25. A spring-balance is carried in a balloon which is ascending vertically. Find the acceleration of the balloon when a half-pound weight hung upon the spring-balance is found to indicate 9 ounces.

26. By what number would the acceleration due to gravity be expressed, if the day and the mile were the units of time and length?

27. If the unit of length were the yard, the unit of time the minute, and the unit of mass a mass of 10 lbs., what would be the value of the unit of force in terms of the poundal?

28. A certain force acts upon a mass of 150 grammes for 10 seconds, and produces in it a velocity of 50 metres per second: compare the force with the weight of a gramme.

29. A force equal to the weight of one pound acts upon a ton: what acceleration is produced, and what will be the velocity at the end of 10 seconds?

30. A certain force acts upon a mass m and generates in it an acceleration a: find the weight which the force would statically support.

31. A spring-balance is graduated at a place where $g = 32{\cdot}3$; at another place where $g = 32$, a body is tested and the balance indicates 8 oz.: what is the correct mass of the body?

32. A certain force acting on a mass of 14 pounds for 10 seconds generates in it a velocity of 128 feet per second. Compare the force with the weight of one pound, and determine the acceleration which it would produce in a mass of one ton.

33. An 18-ton truck is moving at the rate of 30 miles per hour: what is its momentum? (Take the foot and pound as the units of length and mass.)

34. Compare the momentum of a 15-lb. cannon-ball moving at the rate of 300 feet per second, with that of a 3-oz. bullet which has a velocity of 700 yards per second.

35. In what time will a velocity of 45 miles per hour be generated in a train of 80 tons, if the engine exerts upon it a pull equal to a weight of 2 tons?

36. If a body of mass 10 kilogrammes be acted upon for one minute by a force which can statically support 125 grammes, what momentum will it acquire?

37. A body of mass 4 lbs. is observed to be moving at a rate of 8 feet per second; at this instant a constant force begins to act upon it in the direction of its motion, and after 20 seconds its velocity has increased to 24 feet per second. Determine the magnitude of the force, and explain clearly what unit of force you employ in your solution.

38. Compare the amounts of momentum in (1) a 56-lb. weight which has fallen for 2 seconds from rest, and (2) a cannon-ball of 12 lbs. moving with a velocity of 900 feet per second.

39. If the mile be taken as the unit of length, and the acceleration caused by gravity as the unit of acceleration, what will be the unit of velocity?

40. A 7-lb. weight hanging over the edge of a smooth table drags a mass of 56 lbs. along it: find the acceleration, and the distance moved through in 5 seconds from rest.

41. A falling weight of 162 grammes is connected by a string to a mass of 1800 grammes lying on a smooth flat table: find the acceleration, and the tension of the string.

42. A mass of 3 lbs. is drawn along a smooth horizontal table by a mass of 6 oz. hanging vertically: calculate the space described in 3 seconds.

43. A force of 30 dynes acts for 12 seconds upon a body resting on a smooth horizontal plane, and imparts to it a velocity of 120 centimetres per second: what is the mass of the body?

44. A mass of 15 lbs. lying on a smooth flat table is acted upon by a force of 60 poundals: how far will it move in 6 seconds?

45. A body of mass 10 is connected with another body of mass 6 by a string passing over a frictionless pulley: find the acceleration and the distance moved through in 4 seconds. Show how such an arrangement could be employed for finding the value of g, and explain why the method would be better than that of experimenting with a freely falling body.

46. Weights of 14 and 21 lbs. are hung on the ends of a rope passing over a pulley: find the tension in the rope in pounds weight and in poundals.

47. Two masses of 100 and 120 grammes are attached to the extremities of a string passing over a smooth pulley: if the value of g is 975, what will be the velocity after 8 seconds?

48. Two unequal masses are attached to the ends of a string passing over a smooth peg: find the ratio between them in order that each may move through 16 feet in 2 seconds, starting from rest.

49. Two buckets, each weighing 28 lbs., are suspended from the ends of a rope passing over a windlass; a gallon (10 lbs.) of water is poured into one of the buckets: find how far it will descend in 10 seconds, neglecting friction.

50. Describe Atwood's machine, and explain how it may be used to prove—

(*a*) That when different forces act upon the same mass the accelerations observed are proportional to the forces.

(*b*) That when the force is constant the accelerations are inversely proportional to the masses.

(*c*) That the space described in n seconds from rest is proportional to n^2.

What other experimental method has been devised for testing the last proposition?

51. The sum of the two weights in an Atwood's machine is 2 lbs., and the difference between them is an ounce; find the acceleration and the space described in the first second.

52. In an experiment with Atwood's machine the masses were 520 and 480 grammes; in 2 seconds from rest the heavier mass descended 76 centimetres. What value does this give for the acceleration of gravity? If your result differs from the usual value, suggest any cause for the difference.

53. The two equal masses in an Atwood's machine are each 100 grammes; what excess weight must be placed upon one of them in order that, at the end of 3 seconds, it may be descending with a velocity of 2 metres per second?

54. By means of an Atwood's machine a force equal to the weight of 10 grammes was made to act upon a mass of 500 grammes, and it was found that an acceleration of 19·6 cm. per second was produced. Find the value of g.

55. A train starts from rest on a level line and moves through 1200 feet in the first minute. It then begins to ascend an uniform incline, up which it is found to run with uniform velocity: find the inclination of this portion of the line on the supposition that the engine exerts a constant pull.

56. A body of mass m moves with uniform velocity v in a circle of radius r. Prove that a force $\frac{mv^2}{r}$ is required to keep it in its circular path, and that this force is directed along the radius and towards the centre.

57. A body of mass 2 lbs. is attached to the end of a string a yard long, and is whirled round at an uniform rate, making twenty revolutions in a minute: what is the tension in the string?

58. A mass of a kilogramme is connected to a fixed point by a string one metre in length, and whirls round in a circle once a second: find the tension of the string n terms of the weight of a gramme.

59. Assuming that there are 86,164 seconds in a sidereal day, and that the earth's mean equatorial radius is 3962 miles; calculate (in feet per second) the acceleration of a point on the equator.

60. Starting from the result of the preceding problem, discuss the effect of the earth's rotation upon a spring-balance which is used to weigh the same body (1) at the pole, (2) at the equator; and show that if the earth revolved about seventeen times as fast as it now does, a body on the equator would have no apparent weight.

61. Prove, by any method, that the time of a complete oscillation of a simple pendulum is $2\pi\sqrt{\frac{l}{g}}$ when the amplitude of oscillation is indefinitely small.

62. Find the value of g at a place where the length of the seconds pendulum is 0·994 metre.

[*N.B.*—A seconds pendulum is one which makes half a complete oscillation in a second.]

3 m

63. A pendulum 10 feet in length makes ten complete oscillations in 35 seconds: what is the value of g at the place?

64. Supposing a pendulum to be constructed to beat seconds at a place where $g = 950$; how would its length have to be altered in order to make it beat seconds on the surface of the moon, where $g = 150$?

65. Show that a pendulum of one metre in length would beat seconds if the value of g were 987.

66. What is the value of g at Greenwich, where the length of the seconds pendulum is found to be 39·14 inches, and what is the length of a pendulum which loses 10 minutes a day at this place?

67. The bob of a pendulum can be raised by means of a screw which has thirty threads to the inch; if the pendulum loses 5 minutes a day, how many turns of the head of the screw must be made in order to correct it?

(Assume that the pendulum keeps correct time when its length is 39 in.)

68. Enunciate the law of universal gravitation, and give an account of the method of measuring the attraction between two spheres, devised by Mitchell, and carried out by Cavendish.

69. How did Newton prove that the weight of a body is proportional to its mass? Describe the nature of his experiment, and explain how he deduced his conclusions.

70. Assuming the preceding proposition, and the third law of motion, show that it follows immediately that the attraction between two gravitating masses is directly proportional to the product of these masses.

71. Prove that a spherical shell exerts no attraction upon a particle placed within it. You may assume

(*a*) That the area of a transverse section of a cone of small aperture varies as the square of the distance from the vertex.

(*b*) That a transverse section has a smaller area than an oblique section at the same distance, in the proportion of the cosine of the angle between them.

72. Prove that a uniform spherical shell attracts an external particle as if its whole mass were condensed at its centre.

Work.—When the point of application of a force F moves through a distance s in the direction of the force, the work (W) done is

$$W = Fs.$$

If the force is measured in *dynes* (see § 3) and the distance in *centimetres*, the work will be expressed in *ergs*.

If the force is measured in poundals and the distance in feet, the work will be expressed in foot-poundals. (A poundal, or British absolute unit of force, is that force which, acting upon a mass of one pound, generates in it an acceleration of one foot per second in a second.)

When the unit of force is one which depends upon the intensity of gravity, the work is expressed in gravitation-units, whose value varies from place to place. By way of distinction, the erg and the foot-poundal are called absolute units. The engineer's unit of work—the foot-pound—is a gravitation-unit; it represents the work done at any place in raising a pound weight vertically through a distance of one foot at that particular place. A foot-pound is equal to g foot-poundals; or, taking $g = 32$,

$$1 \text{ foot-pound} = 32 \text{ foot-poundals.}$$

Since half an ounce is $\frac{1}{32}$ of a pound, the foot-poundal about corresponds to the work done in raising half an ounce through a vertical distance of one foot.

The kilogramme-metre (which is the French engineer's unit of work) is the work done in raising the weight of a kilogramme through a vertical distance of one metre against the force of gravity. It is open to the same objections as the foot-pound, viz. that its value varies from place to place and from level to level.

The gramme-centimetre is the work done in raising a gramme weight through a vertical distance of one centimetre; at a place where $g = 981$, the weight of a gramme is 981 dynes, and a gramme-centimetre is equal to 981 ergs.

73. Find the work done by a force of 50 dynes acting through a distance of 2 metres.

$$\text{Here } F = 50,$$
$$s = 2 \text{ metres} = 200 \text{ cm.}$$

The work done is

$$W = Fs = 50 \times 200 = 10{,}000 \text{ ergs.}$$

74. How much work is done in raising a weight of one ton through a vertical height of 5 yards?

$$1 \text{ ton} = 2240 \text{ lbs.},$$
$$\text{and } 5 \text{ yards} = 15 \text{ feet.}$$

Expressed in foot-pounds, the work done is

$$2240 \times 15 = 3360.$$

Expressed in foot-poundals, the work is

$$2240 \times 15 \times 32 = 107520 \text{ (taking } g = 32).$$

75. The weight of a tram-car is 8 tons, and the resistance due to friction encountered in moving it is equal to one-sixteenth of the weight of the car: how much work is done in a run of 4 miles?

$$\text{The resistance to motion} = \text{weight of } \tfrac{8}{16} \text{ ton,}$$
$$= \text{weight of } 1120 \text{ pounds.}$$

The distance through which this force is overcome is

$$4 \text{ miles} = 4 \times 1760 \times 3 \text{ feet,}$$
$$= 21120 \text{ feet.}$$

The work done (expressed in gravitation-units) is

$$1120 \times 21120 \text{ foot-pounds} = 23,654,400 \text{ foot-pounds.}$$

76. Assuming that a person walking on level ground does work equivalent to the raising of his own weight vertically upwards through one-twentieth of the distance walked, find (in foot-tons) the average daily work done by Weston in his walk of 5000 miles in 100 days, his weight being 9 stone 2 lbs.

The average daily walk was 50 miles, and the average daily work was equivalent to the raising of his own weight through

$$\frac{50}{20} \text{ miles} = \frac{50 \times 1760 \times 3}{20} \text{ feet} = 50 \times 88 \times 3 \text{ feet.}$$

The weight raised was 9 stone 2 lbs. = 128 lbs. Hence the average daily work, in foot-tons, was

$$\frac{50 \times 88 \times 3 \times 128}{112 \times 20} = \frac{5280}{7} = 754\frac{2}{7}.$$

77. A mass of 12 kilogrammes is raised through a vertical height of 8 metres: express the work done in gramme-centimetres, and convert this into ergs.

78. A man can pump 30 gallons of water per minute to a height of 16 feet: how many foot-pounds of work does he do in an hour?

79. An agent A exerts a force equal to a weight of 50 lbs. through a distance of 120 feet, and another agent B exerts a force equal to a weight of 180 lbs. through 90 feet. What is the ratio between A's work and B's?

80. A ladder 20 feet long rests against a vertical wall and is inclined at an angle of 30° to it: how much work is done by a man weighing 13 stone in ascending it?

81. A body of mass 3 lbs. is projected vertically upwards with a velocity of 640 feet per second: how much work has been done against gravity when it has ascended to half its maximum height?

82. How much work would be done in lifting 8 kilogrammes to a height of 12 metres above the surface of the moon, where g is 150?

83. Two masses M and M^1 are acted upon by the same force for the same time: find the relation between (1) the amounts of momentum generated in the masses, (2) the amounts of work done upon them.

84. A body of mass 12 lbs. rests upon a horizontal plane, the coefficient of friction between it and the plane being 0·14: find the work done in moving the body through a distance of 4 yards along the plane.

85. If the plane in the preceding question were inclined at an angle of 30° to the horizontal, how much work would have to be done in order to move the body 3 yards along it?

86. Calculate, in foot-tons, the work done in moving a railway train weighing 120 tons through a distance of 2 miles along a level line, assuming that the resistances amount to 12 lbs. for every ton in motion.

87. If we change from a foot-pound-second to a yard-pound-minute system, in what ratio must we alter the unit of work?

88. Find the work done in drawing a carriage of 20 tons up an incline one mile in length and rising 1 in 120, the coefficient of friction being $\frac{1}{140}$.

89. The cylinder of a steam-engine has a diameter of 6 inches, and the piston moves through a distance of 10 inches: find the work done per stroke, assuming the pressure of the steam in the cylinder to be constant and equal to 30 lbs. per square inch.

90. Two bodies of 80 lbs. and 60 lbs. are raised through heights of 100 feet and 50 feet respectively. Calculate the total amount of work done, and show that it is equal to the work which would be done in raising the sum of the weights through a vertical distance equal to that through which their centre of gravity is raised.

91. Assuming the result indicated in the preceding question, and taking the weight of one cubic foot of water as 62·5 lbs.; find how much work must be done in order to empty a well 10 feet in diameter and 200 feet deep, filled to the brim with water.

Energy.—The energy of a body is the power which it possesses of doing work. When it possesses this power in virtue of its position (as when it is raised above the level of the ground) the energy is called statical or potential energy. When it possesses the power of doing work in virtue of its motion, the energy is called kinetic energy (K.E.)

The weight of a body of mass m grammes is mg dynes; if the body be raised to a height of h centimetres above the level of the ground, the work which it can do in falling is mgh ergs, or

The potential energy of a body of mass m, raised to a height h, is mgh.

If m is expressed in pounds and h in feet, the product mgh will be the measure of the energy in foot-poundals; but expressed in foot-pounds the measure of the energy will be mh simply.

Now if a body be projected vertically upwards with a velocity v, it will rise to a height h, such that

$$v^2 = 2gh.$$

Multiplying each side of this equation by $\frac{m}{2}$, we have

$$\tfrac{1}{2}mv^2 = mgh.$$

But mgh represents the work which would have to be done in order to raise the body to the height h; and this amount of work is done by the body in virtue of the energy of motion, or kinetic energy, which it possessed on starting; hence

The kinetic energy of a body of mass m moving with a velocity v is $\tfrac{1}{2}mv^2$.

[It should be noticed that since

$$\tfrac{1}{2}(2) \times 1^2 = 1,$$

the unit of kinetic energy is that possessed by *two* units of mass moving with unit velocity (*not* that possessed by unit mass moving with unit velocity.)]

92. A reservoir contains water at a height of 200 feet above the ground: what is the potential energy of the water in foot-pounds per gallon?

The potential energy of each pound of water in the reservoir is 200 foot-pounds, and one gallon of water = 10 lbs. Hence the potential energy per gallon is 10 × 200 = 2000 foot-pounds.

93. What is the potential energy of a mass of 25 kilogrammes raised to a height of 40 metres above the ground?

$$\begin{aligned}\text{Its energy} &= 25 \times 40 \text{ kilogramme-metres,}\\ &= 1000 \text{ kilogramme-metres,}\\ &= 1000 \times 10^5 \text{ gramme-centimetres,}\\ &= 981 \times 10^8 \text{ ergs.}\end{aligned}$$

94. A stone of mass 6 kilogrammes falls from rest at a place where $g = 980$: what will be its kinetic energy at the end of 5 seconds?

The velocity acquired in 5 seconds will be

$$v = g \times 5 = 980 \times 5 = 4900,$$

and since 6 kilos. = 6000 gm.,

$$\text{K.E.} = \tfrac{1}{2} \times 6000 \times (4900)^2 = 7{\cdot}203 \times 10^{10} \text{ ergs.}$$

95. What is the K.E. of a body of mass 16 lbs. moving with a velocity of 50 feet per second?

Expressed in foot-poundals the K.E. of the body is

$$\tfrac{1}{2} \times 16 \times (50)^2 = 20{,}000.$$

Since one foot-pound = 32 foot-poundals, the K.E. in foot-pounds is

$$\frac{20{,}000}{32} = 625.$$

[We might have commenced by *defining* kinetic energy as being the value of the product $\frac{1}{2}mv^2$; then proceeding, as follows, to show that this quantity is a measure of the work which a body can do in virtue of its motion:—

Suppose a body of mass m moving with a velocity u to be acted upon by a force F in the direction of its motion, and let the acceleration produced by this force be $a = \frac{F}{m}$. After the body has moved through a space s its velocity v will be given by the equation

$$v^2 = u^2 + 2as.$$

Multiplying each term in this equation by $\frac{m}{2}$, we have

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mu^2 + mas.$$

But since $F = ma$, $mas = Fs$, and Fs is the work done by the force F acting through the space s. Thus the work done by the force is measured by the increase of kinetic energy which it produces; or,

[K.E. at any time] = [initial K.E.] + [work done by acting force].

If the force acts upon the body in a direction opposite to that of its motion, it resists its motion and diminishes its kinetic energy: the + sign in the equation must be changed to −, and we now have

[K.E. at any time] = [initial K.E.] − [work done against resistance].

Both theorems are expressed in the statement that the work done by the force is equal to the change of kinetic energy which it produces.

One case of the second theorem is of special importance. Suppose the body to be brought to rest by the resistance; the final K.E. = 0,

$$\therefore 0 = [\text{initial K.E.}] - [\text{work done against resistance}],$$

or,

$$[\text{Initial K.E.}] = [\text{work done against resistance}].$$

Thus we have shown that the kinetic energy ($\frac{1}{2}mv^2$) of a body is a measure of its capacity for doing work.]

96. A train is moving at the rate of 15 miles an hour when the steam is cut off. Supposing the resistance due to friction etc. to amount to $\frac{1}{64}$ of the weight of the train, find how far it will travel before it comes to rest.

15 miles an hour = 22 feet per second.

If the mass of the train be m lbs., its kinetic energy (in foot-poundals) is

$$\frac{mv^2}{2} = \frac{m}{2} \times (22)^2.$$

The resistance is equal to the weight of $\frac{m}{64}$ pounds

$$= (m/64) \times g \text{ poundals},$$
$$= m/2 \text{ poundals (taking } g = 32).$$

If the train travels x feet, the work done is $(m/2) \times x$ foot-poundals; and, since it is brought to rest by the resistance, this must be equal to its kinetic energy,

$$\therefore (m/2) \times x = (m/2) \times (22)^2,$$

and

$$x = 484.$$

Thus the train travels 484 feet before coming to rest.

97. A bullet of 100 grammes is discharged with a velocity of 400 metres per second from a rifle, the barrel of which is one metre in length. Calculate the energy of the bullet when it leaves the muzzle, and the mean force exerted by the powder upon it.

The kinetic energy of the bullet is

$$\tfrac{1}{2} \times 100 \times (40{,}000)^2 = 8 \times 10^{10} \text{ ergs.}$$

The energy acquired by the bullet is equal to the work done upon it (by the expansion of the powder) as it travels down the barrel. Let F denote the mean force, in dynes, exerted by the powder; then, since $Fs = \tfrac{1}{2}mv^2$,

$$F \times 100 = 8 \times 10^{10},$$

or

$$F = 8 \times 10^8 \text{ dynes.}$$

98. A girl weighing 6 st. 6¾ lbs. skips 6 inches high fourteen times. Show that the energy thus spent would suffice to stop a thief weighing 13½ stones and running at the rate of 10 miles an hour.

13½ st. = 189 lbs., and 10 miles an hour = 44/3 feet per second.

Thus the kinetic energy of the thief (in foot-poundals) is

$$\tfrac{1}{2} \times 189 \times (44/3)^2 = \tfrac{1}{2} \times 21 \times 11^2 \times 4^2.$$

Again the work done by the girl is

$$14 \times [\text{work done in raising } 90\tfrac{3}{4} \text{ lbs. through } \tfrac{1}{2} \text{ foot}]$$
$$= 14 \times \tfrac{1}{2} \times 363/4 \text{ foot-pounds,}$$

or, in foot-poundals (taking $g = 32$),

$$= 7 \times 363 \times 8 = 7 \times 3 \times 11^2 \times \tfrac{1}{2} \text{ of } 4^2 = \text{K.E. of thief.}$$

99. Express (1) in foot-pounds and (2) in foot-poundals the potential energy of a mass of 5 tons raised to a height of 10 yards above the ground.

100. A sack of flour weighs 2½ cwt.: to what height must it be raised in order that its potential energy may be 9240 foot-pounds?

101. A 4-oz. bullet is projected vertically upwards with a velocity of 800 feet per second: what is its potential energy when it has ascended to its maximum height?

102. A mass of 24 kilogrammes is raised to a height of 16 metres: find its energy (in ergs).

103. A kilogramme weight is suspended from the lower end of a string 2 metres long so as to form an approximately simple pendulum: calculate, in ergs, the energy of the bob of this pendulum when it is held so that the string makes an angle of 60° with the vertical.

104. What is the energy of a mass of 5 kilogrammes moving with a velocity of 50 metres per second?

105. A cannon ball of 10 kilogrammes is discharged from a gun with a velocity of 300 metres per second: express its kinetic energy in ergs.

106. Calculate the momentum and the K.E. of a mass of 5 cwt. after it has fallen through a vertical distance of 8 feet.

107. A stone of mass 3 lbs. is thrown vertically upwards with a velocity of 96 feet per second: what is its kinetic energy at the end of 2 seconds?

108. A 5-lb. stone is thrown vertically up and at the end of the first second is moving upwards at a rate of 64 feet per second: calculate its kinetic energy at the moment when it reaches the ground.

109. A mass of 50 lbs. starts from rest under the action of a constant force and acquires a velocity of 12 feet per second in 2 seconds: what force acts upon it, and what will be the kinetic energy acquired at the end of the fifth second?

110. A 100-gramme bullet strikes an iron target with a velocity of 400 metres per second, and falls dead: how much kinetic energy is lost?

111. A body of mass m is moving with a velocity such that its K.E. is e; show that its momentum is $\sqrt{2me}$.

112. A mass of 50 kilogrammes starts from rest under the action of a force, and some time afterwards is observed to be moving with a velocity of 10 metres per second: how many ergs of work have been done upon it?

113. How many foot-pounds of work must be done on a mass of one ton in order to give it a velocity of 15 miles an hour?

114. The mass of a pendulum-bob is 100 grammes, and the string is a metre long. The bob is held so that the string is horizontal, and is then allowed to fall: find its kinetic energy when the string makes an angle of 30° with the vertical.

115. A shot travelling at the rate of 200 metres per second is just able to pierce a plank 4 cm. thick: what velocity is required to pierce a plank 12 cm. thick?

Assuming that the resistance offered by the plank is uniform, it follows from the equation

$$Fs = \tfrac{1}{2}mv^2$$

that the thickness which the shot can penetrate is proportional to the square of its velocity. If a shot moving with velocity v can pierce a plank of thickness t, and a shot moving with velocity v' can pierce a plank of thickness t', then

$$t : t' :: v^2 : v'^2.$$

In the above example

$$v'^2 = (200)^2 \times 12/4 = 120{,}000,$$

and therefore the required velocity is 346·4 metres per second.

116. If a bullet moving with a velocity of 150 metres per second can penetrate 2 cm. into a block of wood, through what distance would it penetrate when moving at the rate of 450 metres per second?

117. What is the energy of a train of 40 tons moving at the rate of 30 miles an hour? What force, acting for 30 seconds, would be sufficient to bring the train to rest?

118. A stone of mass 3 lbs. falls from rest for 2 seconds, when it comes in contact with a flat roofing-slate, which it smashes, thereby losing two-thirds of its velocity: how much energy does it lose by breaking the slate?

119. Compare the amount of kinetic energy in (1) a boulder of one hundredweight which has fallen for one second from rest, and (2) a one-pound projectile moving with a velocity of 800 feet per second.

120. A bullet of 90 grammes leaves the muzzle of a gun with a velocity of 500 metres per second. If the barrel be 120 centimetres long, find the mean pressure exerted by the powder upon the bullet.

121. A railway carriage contains forty passengers, whose average weight is 140 lbs. If the carriage itself weighs 6 tons, and is moving at a rate of 30 miles an hour, what is its kinetic energy?

122. A body of mass m moves under the action of a force F through a space s in a straight line, which is inclined at an angle θ to the direction of the force: if v be the velocity generated, show that $Fs\cos\theta = \frac{1}{2}mv^2$.

123. A 20-lb. cannon ball falls through a vertical distance of 1600 feet: what is its energy? With what velocity would it have to be projected from a cannon in order to possess an equal amount of energy?

124. Two inelastic balls moving in opposite directions come into collision; the one has a mass 10 and velocity 50, the other a mass 50 and velocity 10: what is their total kinetic energy before and after impact, and what has become of the energy apparently destroyed?

125. A body of mass 56 lbs. starts from rest under the action of a constant force, and acquires a velocity of 64 feet per second while moving through a space of 160 feet: find the acting force and the work done by it.

126. A constant force acts upon a body for 20 seconds, doing 10 units of work upon it, and generating in it during the same time 30 units of momentum: find the mass of the body and the velocity which it will have acquired.

127. The bob of a simple pendulum is let go when the pendulum is inclined at an angle of 60° to the vertical. Compare its kinetic energy after describing an arc of 30° with its K.E. at its lowest point.

Power.—The power (or activity) of an agent is the rate at which it can do work, and is measured by the number of units of work done per unit of time. The unit of power commonly used by engineers in this country is the horse-power, which is defined as being the power of doing 33,000 foot-pounds of work per minute, or 550 foot-pounds of work per second.

128. Assuming that the pressure within the cylinder of a steam-engine remains constant throughout the whole of the stroke, find the horse-power developed in each cylinder of an engine, having given—

A = area of piston in square inches.
P = pressure upon the piston in pounds per square inch.
S = length of stroke in feet.
R = number of revolutions per minute.

Here P denotes the *intensity* of the pressure on the piston in pounds weight per square inch. The total pressure on the piston is the weight of AP pounds. This is the acting force, and the distance through which it moves in each stroke is S feet.

Thus the work done in each stroke is SAP foot-pounds.

Since there are two strokes for each revolution, the number of strokes per minute is $2R$, and the work done per minute is $2SRAP$ foot-pounds. Thus the horse-power developed is

$$\text{H.P.} = 2SRAP/33,000.$$

129. Water is supplied to a hydraulic motor at a pressure of 100 lbs. per square inch. Express the potential energy of the water in the reservoir in foot-pounds per gallon; and calculate the maximum H.P. which can be developed by the motor if the rate of supply is 50 gallons per minute.

Let h = height in feet of the reservoir. The pressure in pounds weight per square foot $= h\rho$, where ρ = number of pounds in a cubic foot of water $= 62{\cdot}5$,

$\therefore h \times 62{\cdot}5/144$ = pressure in pounds weight per square inch $= 100$,

and $\qquad h \times 14400/62{\cdot}5 = 230{\cdot}4.$

Thus the potential energy of one pound of water is 230·4 foot-pounds, and the potential energy of one gallon (or 10 lbs.) is 2304 foot-pounds.

(Notice that since the pressure is supposed constant in the question, we must also assume that the level of the water in the reservoir is kept constant.)

The work done by a supply of 50 gallons per minute is

$$2304 \times 50 \text{ ft.-lbs. per min.}$$

The power developed is

$$\text{H.P.} = 2304 \times 50/33{,}000 = 3\frac{27}{55} = 3{\cdot}49.$$

130. The nominal value of a horse-power is 33,000 foot-pounds per minute. Express this (1) in kilogramme-metres per minute and (2) in ergs per second.

131. A five H.P. engine is employed to pump water from the bottom of a mine 100 feet deep. How many cubic feet of water will it raise in 24 hours? (1 cub. ft. of water $= 62\frac{1}{2}$ lbs.)

132. What should be the indicated H.P. of an engine that is intended to pump 200 gallons of water per minute to a height of 50 yards? (1 gal. = 10 lbs.)

133. A 300 H.P. engine draws a train of 180 tons, the resistance due to friction being 12 lbs. per ton: find its maximum velocity along a level line.

134. Find the H.P. of an engine that should be employed for raising coal from a pit 200 feet deep, the average daily yield being 1782 tons.

135. Find the horse-power exerted by an engine which draws 150 tons up an incline of 1 in 200 at the rate of 15 miles per hour, the resistance due to friction, etc. being equivalent to a weight of 14 lbs. for every ton in motion.

136. Determine the rate at which an engine is working when it drives a train of 150 tons at a rate of 30 miles an hour, the resistance to motion being equal to a weight of 16 lbs. for every ton.

137. The mass of a train is 200 tons, and the resistances to its motion amount to 20 lbs. per ton on a level line: find the horse-power of an engine which can just keep it going at the rate of 45 miles an hour.

138. A steam-engine supplies 1000 houses with 100 gallons of water each, working 12 hours per day: if the mean height to which the water has to be raised is 80 feet, at what rate does the engine work?

139. What alteration would be produced in the unit of work if the units of mass, length, and time were each increased ten-fold? If a horse-power be represented by 550 under the old system, what would be its numerical value in the new?

EXAMINATION QUESTIONS.[1]

140. A particle whose mass is M pounds moves from rest under the action of a force of P units which is constant in magnitude and direction: how far will the particle move in *n* seconds, and what space will it describe in the *n*th second?

If the force be the weight of the body, and the particle traverse 176·99 feet during the 6th second of motion, find the value of "g." Matric. 1882.

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[1] The following abbreviations are used in marking the sources from which the examples at the end of each chapter are taken:—

London University Examinations
Matric. = Matriculation.
Int. Sc. = Intermediate Science (or 1st B. Sc.)
B. Sc. = Final B. Sc.
Prel. Sc. = Preliminary Scientific.

Camb. Schol. = Cambridge Inter-Collegiate Scholarship Examinations.
Camb. B.A. = Cambridge General and Special B.A. Exams.
N. S. Tripos = Natural Science Tripos.
M. Tripos = Mathematical Tripos.
Balliol Coll. = Brakenbury Natural Science Scholarship, Balliol College.
Vict. Int. = Intermediate B.A. and B.Sc. Exams., Victoria University.
Ind. C. S. = Indian Civil Service.

141. Explain fully what is meant by the acceleration of gravity, and show how the value of "g" may be determined by Atwood's machine.

If the acceleration of gravity be represented by unity, and one second be the unit of time, what must be the unit of length? Int. Sc. 1883.

142. The effect of force on matter being either *strain* or *change of* motion, show that either effect may be used to measure force. Which is the more convenient measure, and why? Owens Coll. 1881.

143. Prove that when a foot and a minute are taken as units of length and time, the same acceleration is expressed by ten times the number required to express it when the inch and $\sqrt{30}$ seconds are taken as the units. Owens Coll. 1880.

144. Enunciate Newton's second law of motion, and, assuming that the force of gravity upon a pound is 32·2 poundals, find the force of gravity upon 5 grammes in dynes. Camb. Schol. 1881.

145. What is an absolute unit? Specify the absolute units of force, momentum, and work belonging to the foot-second-pound system.

In what ratios will these units be changed if the unit of length be increased to a yard, and the unit of time to a minute? Prel. Sc. 1889

146. What is the meaning of the equation $P = mf$?

If the unit of force be equal to the weight of one cwt., and the unit of acceleration be equal to half a foot-second unit of acceleration, find the number of pounds in the unit of mass. Mason Coll. 1884.

147. Explain the meanings of the terms force, momentum, impulse, energy.

A cannon-shot of 1000 lbs. strikes directly a target with a velocity of 1500 feet per second, and comes to rest: what is the measure of the impulse? If the cannon-shot rebounded with a velocity of 200 feet per second, what would be the measure of the impulse?

Matric. 1883.

148. Two weights, of 5 pounds and 7 pounds respectively, are fastened to the ends of a cord passing over a frictionless pulley supported by a hook. Show that when they are free to move the pull on the hook is equal to $11\frac{2}{3}$ pounds weight. Matric. 1886.

149. A jet of water is projected against a fixed wall so as to strike it at right angles. If the velocity of the jet be 80 feet per second, and 100 lbs. of water strike the wall in each second, what pressure will be exerted against the wall (1) when the water does not rebound; (2) when it rebounds with a velocity of 10 feet per second? Int. Sc. 1883.

150. What is the dynamical unit of force? Show that the unit of force when 10 feet, 100 seconds, and 1000 pounds are the fundamental units is equal to that when n feet, n seconds, and n pounds are the corresponding units. Camb. Schol. 1882.

151. Describe Atwood's machine. From the following data find the numerical value of g, the acceleration due to gravity, at a certain place:—

Mass of each box	30 oz.
Equivalent for inertia of wheel-work .	10 ,,
Mass added to one of the boxes .	2 ,,
Space described from rest in 3 secs. .	4·0 ft.

Glasgow M.A. 1882.

152. Two scale-pans, each weighing 2 oz., are suspended by a weightless string over a smooth pulley. A mass of 10 oz. is placed in one and 4 oz. in the other: find the tension of the string and the pressure on each scale-pan. Matric. 1883.

153. In Atwood's machine one weight is double the other; the wheel is held at rest whilst the string is nailed to the wheel at the top: if the weights be allowed to move gently to their position of equilibrium, show that the line joining the nail to the axle will make an angle of 30° with the vertical.

Also, if the weights be, instead, allowed to move freely after the nailing, find the kinetic energy of the weights and wheels when the nail arrives at this position. Balliol Coll. 1881.

154. If two weights w and $2w$ are connected by a string passing over a smooth weightless pulley, which is attached to a third weight $3w$ by a string passing over a smooth fixed pulley, prove that the weight $3w$ descends with an acceleration $\frac{g}{17}$. Camb. Schol. 1882.

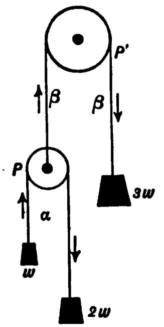

Fig. 1.

Let α denote the acceleration of the weight $2w$ downwards *relative to the pulley P*, and let β denote the acceleration of P upwards, or of the weight $3w$ downwards.

The actual acceleration of $2w$ in space (or relative to the fixed pulley P′) is $\alpha - \beta$. Let τ be the tension in the string connecting w and $2w$. Considering the motion of $2w$, we see that the force causing motion is $2wg - \tau$, and the mass moved is $2w$, and the acceleration produced is $\alpha - \beta$.

Therefore

$$2w - \tau = 2w(\alpha - \beta).$$

The acceleration of w upwards is $\alpha + \beta$, and the force acting upon w is $\tau - wg$; hence

$$\tau - wg = w(\alpha + \beta).$$

The tension in the string connecting the third weight with the pulley P is $\tau' = 2\tau$, and the equation for the motion of $3w$ is

$$3wg - 2\tau = 3w\beta.$$

Solving these three equations, we find that the weight $3w$ descends with an acceleration $g/17$.

155. Taking the earth to be a sphere of radius 4000 miles, rotating about its axis in $88,000^s$ roughly, and assuming $\pi^2 = 10$, $g = 32$ feet per second at the pole, show that the weight of a body will be decreased by about $\frac{1}{290}$ of the whole by taking it from the pole to the equator. Balliol Coll. 1881.

156. What is erroneous in the term *centrifugal force?*

A skater describes a circle of radius 100 feet with a velocity of 18 feet per second: what is his inclination to the ice? Edinb. M.A. 1884.

157. Investigate the relation between the length of a simple pendulum and the time of oscillation. A "second's" pendulum is lengthened by 1 per cent: how much will it lose in a day? ($g = 32$.)

Glasgow M.A. 1882.

158. What is meant by a "simple pendulum," and "equivalent simple pendulum"?

A pendulum vibrates seconds at a certain place; it is taken to another where the value of g is greater by 1 per cent.: find the number of vibrations greater or less per day. Glasgow M.A. 1884.

159. A pendulum 37·8 inches long makes 183 beats in 3 minutes at a certain place: find the force of gravity there. Ind. C. S. 1886.

160. What is the meaning of the expression "the length of the seconds-pendulum"? How far could it be rational to make that length, or some sub-multiple of it, the unit of length? What would be the acceleration of gravity if the length of the seconds-pendulum were the unit of length? N. S. Tripos. 1884.

161. A balloon ascends with a constant acceleration, and reaches a height of 900 feet in one minute. Show that a pendulum clock carried with it will gain at the rate of 27·8 seconds per hour. M. Tripos. 1883.

162. Explain the terms *mass of a pound*, *weight of a pound*, *foot-pound.*

A mass of iron weighing 20 pounds (1) falls freely under the action of gravity ($g = 32$ ft.-sec.); (2) descends with an acceleration of 25 (ft.-sec.) Determine in each case the kinetic energy of the mass in foot-pounds when it has fallen through 100 feet. Int. Sc. 1886.

163. A man weighing 12 stones does 360,000 foot-pounds of work against gravity in ascending a hill: find the height ascended. Glasgow M.A. 1882.

164. Masses of 485 and 496 grammes respectively are connected by a fine string and hung over a very light pulley as in an Atwood machine. Find, by considering the work done, the velocity after a fall of 88 cm. from rest, supposing that friction may be neglected. ($g=981$.)

If the actual velocity is only three-fourths the calculated velocity, find the energy which has been taken up by the friction. Vict. Int. 1885.

165. State the laws of friction.

A curling rink is 42 yards long from tee to tee. A curling stone is hurled from one tee with a velocity of 8·4 feet per second, and comes to rest at the other tee. Find the coefficient of friction between the stone and the ice. Edinb. M.A. 1884.

166. A constant force acting on a body generates in 10 seconds 20 units of momentum, and does in the same time 20 units of work: find the mass of the body and its velocity at the end of 10 seconds.

N. S. Tripos. 1886.

167. While a railway train travels half a mile on a level line, its speed increases uniformly from 15 miles an hour to 30 miles an hour: show what proportion the pull of the engine bears to the weight of the train. (Neglect friction.) Matric. 1885.

168. A bullet of mass 20 grammes is shot horizontally from a rifle, the barrel of which is one metre long, with velocity 400 metres per second, into a mass of 50 kilogrammes of wood floating on water. If the bullet buries itself in the wood without making splinters or causing the wood to rotate, find the velocity of the wood directly after it is struck (that is, before the velocity has been diminished by the resistance of the water). Also find the average force in grammes-weight exerted on the bullet by the powder ($g = 981$ centimetres per second, per second). Int. Sc. 1885.

169. Show that the difference of kinetic energy of two particles, after sliding through the same distance down two planes, equally inclined to the horizon, but one rough, the other smooth, is numerically equal to the work done against friction by the former particle. Owens Coll. 1884.

170. Prove that a train going at 45 miles an hour will be brought to rest in about 284 yards by the breaks, supposing them to press on the wheels with half the weight of the train, and that the coefficient of friction is ·16. Int. Sc. 1885.

171. A cannon-ball weighing 10,000 grammes is discharged with a velocity of 45,000 centimetres per second from a cannon, the length of whose barrel is 200 centimetres: prove that the mean force exerted on the ball during the explosion is $5{\cdot}0625 \times 10^{10}$ dynes.

Camb. Schol. 1884.

172. Calculate the amount of work done against gravity in drawing a car of 2·5 tons weight, laden with thirty passengers averaging 9 stones each in weight, up an incline the ends of which differ 120 feet in level. Find the horse-power sufficient to do that work in half an hour. Edinb. M.A. 1882.

173. A string has one end fastened to a fixed point; it then goes under a pulley which sustains a weight of 2 kilogs., and then over a fixed pulley, and at its end it sustains another weight of 2 kilogs. The strings are all vertical, and the weight of the pulleys and all friction may be neglected. The weight is pulled up by the free weight descending. Find how long it will take thus to raise the weight through 2 metres, and show that 392,000,000 ergs more work can be done by the kinetic energy of the system. Balliol Coll. 1880.

174. The amount of work which can be derived from the consumption of a kilogramme of coke is 309×10^{12} ergs. Of the work thus derived one-twentieth is usefully employed in drawing trucks up a slope of 30°. Find the amount of coke required to draw a weight of 9400 megadynes a distance of 1500 metres along the slope. (1 megadyne = 10^6 dynes.) Ind. C. S. 1885.

175. Define *kinetic energy* and *work*. Calculate the kinetic energy of a tram-car weighing 2·5 tons, when it is moving at the rate of 6 miles an hour, and is laden with thirty-six passengers, averaging 9 stones each in weight.

If the coefficient of kinetic friction for a tram-car moving on its rails is $\frac{1}{8}$; find how much work is done when the above car, loaded as stated, is pulled 3 miles along a level road. Edinb. M.A. 1881.

176. Compare the amounts of momentum and of kinetic energy in (*a*) a pillow of 20 lbs., which has fallen through one foot vertically, and (*b*) an ounce bullet moving at 200 feet per second. Edinb. M.A. 1881.

177. Compare the strength of a locomotive engine that can get up a speed of 20 miles an hour in 2 minutes in a train of 350 tons with that of an engine that can get up a speed of 30 miles an hour in 2 minutes in a train of 250 tons, the line in both cases being level, and friction negligible. Univ. Coll. Lond. 1884.

178. Show that if a body falls freely from rest under the action of gravity, the increase of its kinetic energy during the 10th second is 9·5 g times the increase of momentum during that second, where g is the numerical value of gravity. Camb. Schol. 1883.

179. An engine of one horse-power is capable of doing 33,000 foot-pounds of work per minute. What is the horse-power of an engine which can pump 1000 gallons of water per minute from a well and project it with a velocity of 80 feet per second through a nozzle which is at a height of 40 feet above the surface of the water in the well? Int. Sc. 1882.

180. A railway train of 300 tons is running at 45 miles an hour. Find its energy of motion in foot-tons. Also, if the engine, while getting up the speed of the train, does work on it at an average rate of 100 horse-power, show how long it will take to get up the speed of 45 miles an hour. [Take $g=32$ (feet, seconds); 1 horse-power = 33,000 foot-pounds per minute.]

Int. Sc. 1884.

181. What is the horse-power of an engine which can project 10,000 lbs. of water per minute, with a velocity of 80 feet per second, 20 per cent of the whole work done being wasted by friction, etc.?

[*N.B.*—An agent of one horse-power can do 33,000 foot-pounds of work per minute.]

Matric. 1884.

182. If a bicyclist always works at the rate of one-tenth of a horse-power, and goes 12 miles an hour on the level, prove that the resistance of the road is 3·125 pounds. Int. Sc. 1885.

183. A railway train of mass 100 tons is moving at 20 miles per hour. What horse-power would be required to impart to it this velocity in 5 minutes from starting, in addition to overcoming the resistances supposed uniform and equal to 12 lbs. per ton? (One horse-power = 33,000 foot-pounds per minute.) Camb. Schol. 1884.

CHAPTER II

HYDROSTATICS

IN solving the examples in this chapter, the following facts may be assumed :—

The mass of a cubic foot of water is 1000 ounces or $62\frac{1}{2}$ pounds.

The specific gravity of mercury is 13·6.

In examples on fluid pressure, the pressure of the atmosphere may be neglected, unless the contrary is expressly stated.

The normal atmospheric pressure is that due to a column of mercury 76 centimetres in height. When the inch is taken as the unit of length, the normal barometric height may be assumed as 30 inches.

Geometrical Relations.—The ratio of the circumference of a circle to its diameter is 3·1416 (or approximately $\frac{22}{7}$), and is denoted by the Greek letter π.

The circumference of a circle of radius r is $2\pi r$, and its area is πr^2.

The area of the surface of a sphere is $4\pi r^2$.

The volume of a sphere is $\frac{4}{3}\pi r^3$.

1. Define a fluid, and explain what is meant by the pressure of a fluid at any point within it. Prove that the pressure at any point of a fluid at rest is the same in every direction.

2. State and illustrate the law of transmission of pressures in liquids, and explain how it is applied in the construction of hydraulic presses.

3. Distinguish between pressure and intensity of pressure; and find the dimensions of each of these quantities in terms of the fundamental units of length, time, and mass.

4. Define density and specific gravity, pointing out the essential distinction between them. Explain how it is that the density of a substance has, in the C.G.S. system, the same numerical value as its specific gravity, or density relative to water.

5. A block of mahogany 2 inches long, $1\frac{1}{2}$ inch broad, and $\frac{7}{8}$ inch thick is found to weigh[1] 461 grains: express its density in grains per cubic inch.

The volume of the block is $2 \times \frac{3}{2} \times \frac{7}{8} = \frac{21}{8}$ cubic inches, and its mass is 461 grains. The density, in grains per cubic inch, is

$$\Delta = \frac{\text{mass}}{\text{volume}} = 461 \bigg/ \frac{21}{8} = 175{\cdot}6.$$

6. A ton of clay is found to occupy a volume of 18 cubic feet: what is its density in pounds per cubic foot?

7. The density of water is $62\frac{1}{2}$ pounds per cubic foot: express this in ounces per cubic inch.

8. Basalt is three times as heavy as water: what is its density in ounces per cubic inch?

9. A ton of chalk occupies a volume of $15\frac{1}{2}$ cubic feet: what is its *specific gravity* referred to water as the standard substance?

The mass of a cubic foot of the chalk is $(2240 \div \frac{31}{2})$ pounds, and the mass of a cubic foot of water is $62\frac{1}{2}$ pounds. Hence the required specific gravity (or ratio between the masses of equal volumes) is

$$\left(2240 \times \frac{2}{31}\right) \div \frac{125}{2} = \frac{2240 \times 2 \times 2}{31 \times 125} = 2{\cdot}312.$$

10. A cubic inch of a substance weighs half a pound: what is its specific gravity?

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[1] In this and the succeeding chapters the term weight is occasionally used instead of mass, when there is no danger of confusion between the two. When a body is said to be weighed in air, the weight of the displaced air may be neglected, unless the contrary is expressly stated.

11. The specific gravity of lead is 11·4: what is the weight of a cube of lead 3 inches in the side?

12. A mahogany block of the same dimensions as that in Example 5 is found to weigh 30·35 grammes: express its density in grammes per cubic centimetre.

13. The specific gravity of iron is 7·7: what is the weight of an iron rod $2\frac{1}{2}$ inches broad, 2 inches thick, and 18 feet long?

14. A body weighs 1000 pounds, and its density is five times that of water: what is its volume?

15. How many grammes of mercury will be required to fill a cylindrical glass tube, the length of which is 70 centimetres, and internal diameter 0·8 centimetre?

The cross-section of the tube is $\pi r^2 = \frac{22}{7} \times (0{\cdot}4)^2 = \frac{22}{7} \times 0{\cdot}16$ square centimetre, and its internal volume is $70 \times \frac{22}{7} \times 0{\cdot}16 = 35{\cdot}2$ cubic centimetres. The specific gravity of mercury is 13·6 approximately, and this number also represents its density in the C.G.S. system: *i.e.* 1 cubic centimetre of mercury = 13·6 grammes.

Thus $35{\cdot}2 \times 13{\cdot}6 = 478{\cdot}72$ grammes are required to fill the tube.

16. A tube 120 centimetres long holds 600 grammes of mercury: find its cross-section and internal diameter.

17. Find the mass of a piece of copper wire 5 metres in length and 1·8 millimetre in diameter, given that the density of copper is 8·8.

18. A cylindrical tube 1 metre in length and 1 centimetre in internal diameter weighs 100 grammes when empty, and 210 grammes when filled with a liquid: find the specific gravity of the liquid.

19. Find the diameter of a cylindrical kilogramme weight made of brass (density 8·4), its height being 7·5 centimetres.

20. The radii of two spheres are 2 centimetres and 3 centimetres, and their masses are 200 grammes and 250 grammes respectively: compare their densities.

21. A litre of hydrogen gas weighs 0·0896 gramme, and the density of carbon-dioxide is twenty-two times that of hydrogen: how much carbon-dioxide is required to fill a gas-bag which holds 10 litres?

22. One litre of a liquid of specific gravity 1·4 is mixed with 2 litres of a liquid of specific gravity 0·96, and the mixture occupies nine-tenths of the volume of its components: what is its specific gravity?

The volume of the mixture is

$$V = \frac{9}{10} \times 3000 = 2700 \text{ c.c.}$$

If Δ denote its density, then its mass in grammes is

$$V\Delta = (1000 \times 1{\cdot}4) + (2000 \times 0{\cdot}96) = 1400 + 1920 = 3320.$$

Thus

$$\Delta = \frac{3320}{2700} = 1{\cdot}23.$$

23. If the specific gravities of two liquids be 4 and 5 respectively, find the specific gravity of a mixture containing 3 parts by weight of the former to 4 parts of the latter.

24. What would have been the specific gravity of the mixture in the last example if the proportions had been taken by volume instead of by weight?

25. Equal volumes of two liquids whose specific gravities are s and $2s$ are mixed together, and the mixture occupies four-fifths of the sum of the volumes of its components: what is its specific gravity?

26. The density of fire-damp is one-half that of air: what is the density of foul air containing 15 per cent of fire-damp?

27. Equal weights of two liquids whose specific gravities are 0·9 and 0·7 are mixed together, and a contraction of 10 per cent occurs in the volume: what is the specific gravity of the mixture?

28. Equal volumes of three fluids are mixed. The specific gravity of the first is 1·55, that of the second 1·75, and that of the mixture is 1·6: find the specific gravity of the third.

Fluid Pressure.—At a depth z below the surface of a heavy liquid of density ρ the pressure is

$$p=g\rho z,$$

neglecting atmospheric pressure.

When g, ρ, and z are all expressed in the C.G.S. system, p denotes the pressure in dynes per square centimetre; if the factor g be omitted, p will represent the pressure in grammes weight per square centimetre. What we have here called pressure is really intensity of pressure, or pressure per unit area; and the pressure on a surface of area a is $P=pa$, if the intensity of the pressure over the surface is uniform.

Taking into account the pressure due to the atmosphere, the pressure at a depth z is

$$g\rho z+\Pi,$$

where Π denotes the atmospheric pressure on unit area of the surface of the liquid.

29. Find the pressure due to a column of mercury 1 metre high.

Here $z=100$, $\rho=13.6$, and taking $g=981$, we have

$$\begin{aligned} p&=981\times 100\times 13.6\\ &=1{\cdot}334\times 10^6 \text{ dynes per sq. cm.}\\ &=1{\cdot}334 \text{ megadynes per sq. cm.}\end{aligned}$$

Expressed in grammes weight per square centimetre the pressure would be $100\times 13{\cdot}6=1360$.

30. What is the pressure at a depth of 50 feet below the surface of the sea, the specific gravity of sea-water being 1·025?

The pressure in pounds per square foot is $p=\rho z$, where

ρ = density of sea-water in lbs. per cub. ft.,
and z = depth below the surface in feet.

Now 1 cubic foot of water weighs 62½ pounds, and therefore 1 cubic foot of sea-water weighs $62{\cdot}5\times 1{\cdot}025=64{\cdot}06$ pounds; thus the pressure is $50\times 64{\cdot}06=3203$ pounds per square foot.

31. Find the pressure due to a column of water 1 metre in depth. Express your result (1) in grammes weight per square centimetre, and (2) in dynes per square centimetre.

32. Calculate the total pressure (in grammes weight) upon the base of a cylindrical vessel one decimetre in diameter, filled with mercury to a height of 40 centimetres.

33. What must be the height of a column of mercury to exert a pressure of 1 kilogramme per square centimetre?

34. The specific gravity of sea-water is 1·025; calculate the pressure in grammes weight per square centimetre at a depth of 40 metres below the surface of the sea.

35. Mercury is poured into a vessel until the layer is 10 centimetres deep, and then water is added until the depth of the water-column is 75 centimetres: determine the pressure on the base in dynes per square centimetre.

36. What must be the height of a column of water in order that the pressure at its base may be a megadyne (10^6 dynes) per square centimetre?

37. Find the equivalent in dynes per square centimetre of a pressure of 1000 kilogrammes weight per square metre.

38. Determine the pressure in grammes weight exerted upon a horizontal area of 2 square decimetres sunk to a depth of 75 centimetres below the surface of oil of specific gravity 0·85.

39. If the inch be the unit of length, and the second the unit of time, what must be the density of the standard substance in order that the equation $p=g\rho z$ may give the pressure in pounds weight?

40. Express in pounds weight per square foot the pressure at the bottom of a lake 300 feet deep.

41. Determine the available water-pressure (in pounds per square inch) in a laboratory which is supplied from a tank at a height of 40 feet.

42. To what depth must a surface be sunk in water in order that the pressure upon it may be 60 pounds per square inch?

43. If a cubic foot of sea-water weighs 64 pounds, what is the pressure at a depth of a mile under the surface of the sea?

44. If the atmospheric pressure be 15 pounds per square inch, what is the pressure at the bottom of a pond 30 feet deep?

45. At what depth below the surface of water will the pressure be equal to two atmospheres, if the atmospheric pressure is 1 megadyne per square centimetre?

46. Show that if two liquids which do not mix meet in communicating tubes, their heights above the common surface of separation will be inversely as their densities.

If the heights of the two liquids above the surface of separation are 15 and 18 inches, and the density of the first is 1·08, what is the density of the second?

47. I wish to discover what the water pressure at a particular tap is, and, in order to do so, I connect it with a mercury manometer; on opening the tap the mercury is forced to a height of 110 centimetres: express the "head of water" in feet.

48. Calculate the available water pressure in pounds per square inch in Example 47.

49. A uniform U-tube is about half filled with water: how much oil of specific gravity 0·8 must be poured into one limb in order to make the water rise 4 inches in the other?

50. The limbs of a V-tube are inclined at an angle of 60°. A length l of the tube is filled with water; it is then held upright with its limbs equally inclined to the vertical, and as much oil of specific gravity s is poured into one side as fills a length l' of it: find the difference of level of the liquids in the two sides.

[In Examples 51-55 the result is to be expressed in grammes weight, neglecting atmospheric pressure.]

51. What is the pressure on an area of 10 square centimetres immersed in water, the centre of gravity of the area being at a depth of 15 centimetres?

52. Calculate the pressure on a circular disc 16 centimetres in diameter immersed in mercury, the centre of the circle being at a depth of 25 centimetres below the surface.

53. A rectangular plate 30 centimetres long and 10 centimetres broad is immersed horizontally at a depth of 3 metres in brine of density 1·1 : what is the pressure upon its surface?

54. What would be the pressure upon the plate in the preceding example, if it were held in a vertical plane with its lower and higher corners 60 and 50 centimetres respectively below the surface?

55. A cube, the edge of which is one decimetre, is suspended in water with its sides vertical and its upper surface at a depth of 1 metre below the surface: find the pressure on each of its faces.

56. Prove that if a cubical box be filled with water, the total pressure to which it is subjected is equal to three times the weight of the water which it contains.

57. Prove that a body immersed in a liquid sustains an upward pressure which is equal to the weight of the liquid displaced: and describe any practical applications of this fact.

58. Describe how you would demonstrate experimentally the truth of the principle of Archimedes; and explain what is meant by the *apparent weight* of a body in water.

A body weighs 62 grammes in vacuo and 42 grammes in water: find its volume and specific gravity.

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59. What will be the apparent weight in water of a piece of rock-crystal (density 2·7) which weighs 35 grammes in vacuo?

60. A bar of aluminium (density 2·6) weighs 54·8 grammes in vacuo: what will be the loss of weight when it is weighed in water?

61. An irregular solid is found to weigh 98 grammes in vacuo and 64 grammes in water: what is its volume?

62. A solid cube, 4 inches in the side, is formed of a substance of specific gravity 12·5: what will its apparent weight in water be?

63. A body which weighs 24 grammes in air[1] is found to weigh 20 grammes in water; what will be its apparent weight in alcohol of specific gravity 0·8?

64. A body which weighs 35 grammes in air is found to weigh 30 grammes in one fluid and 25 grammes in another: what will be its weight when immersed in a mixture containing equal volumes of the two fluids?

65. Two bodies are in equilibrium when suspended in water from the arms of a balance: the mass of the one body is 28 and its density is 5·6; if the mass of the other is 36, what is its density?

The volume of the first body is

$$v_1 = \frac{28}{5\cdot 6} = 5,$$

and if d_2 be the density of the second body, its volume is

$$v_2 = \frac{36}{d_2}.$$

When immersed in water, the apparent weight of the first body is

$$28 - 5 = 23,$$

and of the second

$$36 - \frac{36}{d_2}.$$

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[1] See note on p. 55.

Since these are equal,

$$23 = 36 - \frac{36}{d_2},$$

$$\text{and} \therefore d_2 = \frac{36}{13} = 2{\cdot}77.$$

66. Two masses m_1 and m_2 balance each other when weighed in water; the specific gravity of the one being s_1, what is that of the other?

67. A piece of silver (specific gravity = 10·5) weighing 20 grammes and a piece of tin (specific gravity = 7·3) are fastened to the two ends of a string passing over a pulley, and hang in equilibrium when entirely immersed in water: determine the weight of the tin.

68. What should be the weight of the piece of tin in the preceding example, in order that both bodies might hang in equilibrium when immersed in a liquid of specific gravity 1·5?

69. A string is passed over a pulley so that one end hangs in a beaker of water and the other in a beaker containing salt solution of specific gravity 1·281. A lump of copper (specific gravity 8·9) is hung from one end of the string so as to be immersed in the water: what weight of lead of specific gravity 11·4 must be attached to the other end of the string so as to keep the copper in equilibrium when the lead is completely immersed in the salt solution?

70. Find the acceleration with which a stone of specific gravity 2·8 would sink in water, and the time which it would take to get to the bottom of a pool 16 feet deep.

71. A 7-lb. iron weight (specific gravity 7·3) is suspended from one end of an equal-armed lever and immersed in water: what weight must be hung from the other end of the lever in order to counterpoise it?

72. A cube of gutta-percha (specific gravity 0·96), two centimetres in the side, is suspended from the short pan of a hydrostatic balance, and a basin of water is placed beneath: what weight must be placed upon the pan in order that there may be equilibrium when the gutta-percha is completely immersed?

73. State the conditions which must be fulfilled in order that a body may float in equilibrium in a liquid; and show that if a homogeneous body of volume v and specific gravity s floats in a liquid of specific gravity s', the volume of the part immersed will be $\frac{v \cdot s}{s'}$.

74. What volume of water will be displaced by a kilogramme of wood of specific gravity 0·75 floating in equilibrium?

75. A piece of cork of mass 300 grammes and density 0·25 is placed in a vessel full of water: how much water will overflow?

76. The specific gravity of ice is 0·918 and that of sea-water is 1·03: what is the total volume of an iceberg which floats with 700 cubic yards exposed?

XX

77. A sphere of density 0·95 and volume 100 cubic centimetres floats on water. Oil of density 0·9 is poured upon the water, the layer of oil being deep enough to cover the sphere: how much of it will now be immersed in the water?

78. A solid body floats in water with just half its volume immersed. When it floats in a mixture of equal volumes of water and another liquid, one-third of it is immersed: find the specific gravities of the solid and liquid.

79. A sphere of glass (density 2·8) is dropped into a vessel containing mercury and water: find its position of equilibrium.

80. A solid weighs 100 grammes in air and 64 grammes in a liquid of specific gravity 1·2: what is its specific gravity?

81. A body weighs 124 grammes in vacuo, 108 grammes in water, and 98 grammes in another liquid: what is the specific gravity of the liquid?

82. A solid which weighs 120 grammes in air is found to weigh 90 grammes in water and 78 grammes in a strong solution of zinc sulphate: what is the specific gravity of the solution?

83. A specific gravity bottle weighs 14·72 grammes when empty, 39·74 grammes when filled with water, and 44·85 grammes when filled with a solution of common salt: what is the specific gravity of the solution?

84. A specific gravity bottle when filled with water weighs 61·485 grammes, and when filled with methylated spirit 53·462 grammes. If the bottle weighs 15·063 grammes what is the specific gravity of the spirit?

85. In a determination of the specific gravity of a saturated solution of zinc sulphate with the same bottle, the weight of the bottle filled with water was 61·460 grammes, and, when filled with a solution at 14·6°C., 81·559 grammes: find the specific gravity of the solution at this temperature.

86. Find the specific gravity of a mixture of methylated spirit and water from the following data:—

Weight of empty bottle = 15·056 gm.
„ bottle + water = 61·500 „
„ bottle + mixture = 57·450 „

87. A specific gravity bottle was found to weigh 39·74 grammes when filled with water. Some iron nails weighing 8·5 grammes were introduced, and the bottle was again filled up with water. The weight now was 47·12 grammes: show that the specific gravity of the iron nails was 7·59.

88. After removing the nails from the bottle, 40·37 grammes of lead shot was introduced, and after filling up with water, the weight of the whole was 76·54 grammes: what was the specific gravity of the shot?

89. Find the specific gravity of monoclinic sulphur from the following data:—

Weight of sulphur taken . . .	0·5260	gm.
„ sp. gr. bottle + water (full)	49·9598	„
„ bottle + sulphur + water .	50·2158	„

90. A solid, lighter than water, and weighing 25 grammes in air, is fastened to a piece of metal, and the combination is found to weigh 36 grammes in water: if the piece of metal weighs 45 grammes in water, what is the specific gravity of the light solid?

91. Find the specific gravity of a given solid from the following data:—

Weight of solid in air	0·5	gm.
„ sinker in air	4·0	„
„ solid and sinker in water .	3·375	„
Specific gravity of sinker	8·0	„

92. A sinker weighing 38 grammes is fastened to a piece of cork weighing 10 grammes, and the two together just sink when placed in water: find the specific gravity of the sinker, taking that of cork as 0·25.

93. In order to determine the specific gravity of a piece of fluor-spar, it was placed upon the upper pan of a Nicholson's hydrometer, and weights were added until the instrument sank to a fixed mark in the stem. On removing the fluor-spar an additional weight of 9·85 grammes had to be added. This additional weight was next taken off and the spar placed in the lower pan (immersed in water), when it was found that 3·06 grammes had to be placed in the upper pan to sink the hydrometer to the same mark: what was the specific gravity of the spar?

94. A Nicholson's hydrometer, when floating in water, required a weight of 0·15 gramme to be placed upon the upper dish in order to make it sink to a fixed mark on the stem; and 5·72 grammes had to be placed upon the dish in order to make it sink to the same mark in a solution of salt. If the hydrometer weighed 94·47 grammes, what was the specific gravity of the solution?

95. Walker's specific gravity balance consists of a lever with unequal arms; a fixed weight is hung from the shorter arm, and the longer arm is graduated in inches and tenths of an inch. The lever rests on a knife-edge at X, and is horizontal when a body is suspended in air from a point Y on the long arm; when it is immersed in water the point of suspension has to be moved to Z in order to obtain equilibrium: show that the specific gravity of the body is

$$XZ/(XZ - XY).$$

96. The specific gravity of a piece of solder was determined by Walker's balance. When weighed in air the length of the arm (or distance of the point of suspension from the knife-edge) was 11·05 inches; when weighed in water the length of the arm was 12·55 inches: show that the specific gravity of the solder was 8·37.

97. In order to find the specific gravity of a mineral two experiments were made, the position of the weight on the short arm being altered after the first determination.

		Length of Arm.
Exp. I.	Substance weighed in air	5·78 in.
	,, ,, water	9.07 ,,
Exp. II.	,, ,, air	9·17 ,,
	,, ,, water	14·42 ,,

Show that the mean result of the two experiments is 2·751.

98. The lever of a Walker's balance is in equilibrium when a body is suspended from a point Y on the long arm, the fulcrum being at X. When the body is immersed in water the point of suspension is at Z, and when it is immersed in another liquid the point of suspension is at Z′. Prove that the specific gravity of the liquid is

$$XZ(XZ' - XY)/XZ'(XZ - XY).$$

99. A pebble was suspended from the long arm of the balance, and weighed successively in air, in water, and in methylated spirit, the lengths of the arms being 5·78 inches, 9·1 inches, and 8·27 inches respectively: find the specific gravity of the pebble and of the spirit.

100. The area of the smaller piston of a Bramah press is 2½ square inches, and the area of the larger piston is 200 square inches. A force equal to a weight of 3 pounds is applied to the smaller piston: what pressure will be communicated to the larger one?

101. The lever of a hydraulic press gives a mechanical advantage of 6; the sectional area of the smaller plunger is half a square inch, and that of the larger plunger 15 square inches. A 56-lb. weight is hung from the handle: what weight will the large plunger sustain?

102. If the diameters of the pistons of a Bramah press are in the ratio of 10 to 1, and a power of 14 pounds applied to the smaller piston produces a pressure of 1 ton on the larger, what is the ratio of the arms of the lever used for working the piston?

103. The two pistons of a hydraulic press have diameters of a foot and an inch respectively: what is the pressure exerted by the larger piston when a weight of 56 pounds is placed on the smaller one? If the stroke of the smaller piston is 1½ inch, through what distance will the larger piston have moved after ten strokes?

104. The small piston of a hydraulic press is ¾ inch in diameter, and is worked by a lever: the distance from the piston to the fulcrum is 3½ inches, and from the fulcrum to the power is 35 inches. If the large piston is 9 inches in diameter, what is the mechanical advantage?

105. Give a clear explanation of the manner in which the column of mercury in a barometer is supported. Why is mercury the liquid generally employed? How is the height of the mercurial column affected (1) by changes of temperature; (2) by the narrowness of the tubes; (3) by differences in the value of g?

106. What is meant by the "capacity error" of a cistern barometer? Give a brief description of Fortin's barometer, explaining the adjustment by which the mercury in the cistern is kept at a constant level.

107. How will the reading of a barometer be affected if its tube is not in a vertical position? Calculate, as an example, the length (or apparent height) of the mercurial column in a barometer, the tube of which is inclined at 30° to the vertical, the actual barometric height being 30 inches.

108. What is the height of a water barometer when a mercurial barometer reads 30 inches?

109. During a storm a mercurial barometer falls from 30 to 29 inches: through what distance would a water barometer fall under the same conditions?

110. Find the height of a glycerine barometer when the water barometer stands at 32 feet. (Density of glycerine = 1·27.)

111. When the barometric height is 76 centimetres, a glycerine barometer is found to stand at 810 centimetres. Calculate from this the specific gravity of glycerine referred (1) to mercury; (2) to water, as the standard substance.

112. What would be the height of a barometer containing oil of specific gravity 0·86, when the height of a mercurial barometer is 75 centimetres?

113. Given that the atmospheric pressure is 15 pounds per square inch, calculate its value in kilogrammes per square metre.

Atmospheric pressure per unit area.—If the barometric height h be given, the numerical value of the atmospheric pressure in units of force per unit area may be found as follows :—

Let Π denote the pressure in dynes per square centimetre, and ρ the density of mercury $= 13{\cdot}596$, then

$$\Pi = h\rho g,$$

or, taking the normal barometric height (76 centimetres),

$$\Pi = 76 \times 13{\cdot}596 \times 981 = 1{,}013{,}226.$$

Thus the normal atmospheric pressure is somewhat greater than a megadyne (one million dynes) per square centimetre. Expressed in grammes weight per square centimetre the pressure is

$$\Pi = h\rho = 76 \times 13{\cdot}596 = 1033.$$

This corresponds to a weight of 10,330 kilogrammes per square metre.

Suppose we require to express the atmospheric pressure in pounds weight per square inch, when the barometer stands at 30 inches. Consider a barometer-tube the cross-section of which is 1 square inch; the pressure at the base of the column of mercury is equal to the weight of the column itself. Now we know that 1 cubic foot (or 1728 cubic inches) of water weighs 1000 ounces or 62·5 pounds. Thus 1 cubic inch of water weighs $\frac{62{\cdot}5}{1728}$ pounds, and 1 cubic inch of mercury weighs $\frac{6.25}{1728} \times 13{\cdot}596$ pounds. The weight of a column 30 inches high and 1 square inch in cross-section will therefore be

$$\frac{62{\cdot}5}{1728} \times 13{\cdot}596 \times 30 = 14{\cdot}75 \text{ lbs.}$$

Hence the pressure is equal to a weight of 14·75 pounds on the square inch.

114. Prove that a pressure of 1 megadyne (10^6 dynes) per square centimetre corresponds almost exactly to a barometric height of 75 centimetres.

115. Calculate, in dynes per square centimetre, the atmospheric pressure when the barometer stands at 78 centimetres.

116. What is the atmospheric pressure, in pounds per square inch, when the barometric height is 28·5 inches?

117. Determine the value of the atmospheric pressure in ounces per square foot, when the height of the water barometer is 30 feet.

118. Express in grammes weight per square centimetre, and also in kilogrammes weight per square metre, the pressure due to the atmosphere when the barometric height is 74 centimetres.

119. The mercury in a cistern barometer stands at 75·8 centimetres; calculate, in absolute measure, the pressure on the free surface of the cistern, the area of which is 40 square centimetres.

120. What change in the atmospheric pressure, on a square inch, is indicated by a fall of 1 inch in the height of the barometric column?

121. State Boyle's law, and explain the nature of the limitations to which it is subject. Give an account of Regnault's investigations upon the accuracy of the law.

122. The volume of a quantity of gas is measured when the barometer stands at 72 centimetres, and is found to be 646 cubic centimetres: what would its volume be at the normal pressure?

The normal pressure is that due to a column of mercury 76 centimetres high; and if p and v denote the original pressure and volume, and p' and v' the final pressure and volume, the relation

$$pv = p'v'$$

expresses Boyle's law.

Thus the volume under the normal pressure would be

$$v' = v \times p/p' = 646 \times 72/76 = 612 \text{ c.c.}$$

123. At what pressure would the gas in the preceding question have a volume of 580 cubic centimetres?

The required pressure is given by the equation

$$p' = p \times v/v',$$

and is therefore equal to the pressure due to a column of mercury $72 \times 646/580 = 80{\cdot}19$ centimetres high.

124. A piston is situated in the middle of a closed cylinder, 10 inches long, and there are equal quantities of air on each side of it. The piston is pushed until it is within an inch of one of the ends: compare the pressures on each side.

125. The same quantities of air are contained in two hollow spheres of radii r and r' respectively: compare the pressures within the two spheres.

126. A quantity of hydrogen gas was found to measure 146·8 cubic centimetres when the barometer stood at 78 centimetres. On the next day the volume had increased to 159·2 cubic centimetres: what was now the barometric height?

127. A cylindrical receiver, fitted with a piston, contains air at a pressure of 15 pounds per square inch when the piston is 2 feet from the bottom of the receiver. What will be the pressure of the contained air when the piston is forced down until it is 1 inch above the base: and how far should the piston be pushed in order to increase the pressure to 100 pounds per square inch?

128. Compare the weights of equal volumes of air at pressures of 62·5 centimetres and 81·6 centimetres respectively.

129. A litre of air weighs 1·293 gramme at the normal pressure: find the weight of the air contained in a litre flask when the barometer stands at 82 centimetres.

130. Two closed vessels, A and B, are connected by a stop-cock: A is vacuous, while B contains air at a pressure of six atmospheres. If A is twice as large as B, find the common pressure in both the vessels after the stop-cock is opened.

131. In the preceding question, if A originally contained air at a pressure of 120 centimetres of mercury, and B hydrogen gas at a pressure of 50 centimetres, what would be the pressure after opening the stop-cock?

132. The mouth of a vertical cylinder, 18 inches high, is closed by a piston whose weight is 6 pounds and area 6 square inches. If the piston is allowed to fall, how far will it descend, supposing the atmospheric pressure to be 14 pounds per square inch when the piston is inserted?

133. The same quantities of air are contained in two hollow spheres whose radii are r and r' respectively: compare the whole pressures on the spherical surfaces.

134. A tube, 6 feet in length, closed at one end, is half filled with mercury, and is then inverted with its open end just dipping into a mercury trough. If the barometer stands at 30 inches, what will be the height of the mercury inside the tube?

> The air in the tube expands and depresses the mercury until the pressure of the column of mercury and the pressure of the contained air (diminished by expansion) are together equal to the atmospheric pressure (30 inches). Suppose the mercury to fall until it stands x inches higher inside the tube than in the trough; then the pressure on the air inside is $(30-x)$ inches. The volume of the air is proportional to the length of the tube which it occupies; this was 3 feet = 36 inches when the pressure was 30 inches, but under the diminished pressure of $(30-x)$ inches the length occupied is $(72-x)$ inches. Hence, by Boyle's law,
>
> $$30 \times 36 = (30-x)(72-x).$$
>
> The roots of this quadratic equation are 90 and 12; and on inspection we find that the latter root alone satisfies the conditions of the problem: the root 90 is the solution of a different problem, the algebraical statement of which would lead to the same equation.

135. A glass tube, 1 metre in length, and open at both ends, is plunged into a deep cistern of mercury until a length of 90 centimetres is submerged. The top is now closed by the finger, and the tube is raised until a length of 90 centimetres stands out of the mercury. If the barometric height at the time is 75 centimetres, at what height will the mercury stand inside the tube ?

136. To what height should the tube in the preceding example have been raised in order that the air might occupy a length of exactly 30 centimetres ?

137. The tube of a barometer has a cross-section of 1 square centimetre, and when the mercurial column stands at 77 centimetres the length of the vacuous space above it is 8 centimetres : how far will the mercurial column be depressed if 1 cubic centimetre of air is passed up into the tube ?

Suppose the mercury to be depressed through x centimetres ; then x measures the pressure to which the enclosed air is exposed. Since the section of the tube is 1 square centimetre, the volume of the air under the pressure x centimetres is $(8+x)$ cubic centimetres. Applying Boyle's law,

$$77 \times 1 = x\,(8+x),$$

or

$$x^2 + 8x - 77 = 0,$$

the positive solution of which is $x = 5{\cdot}65$. Thus the mercury is depressed through 5·65 centimetres, and the barometric height, as indicated by this faulty barometer, is now $77 - 5{\cdot}65 = 71{\cdot}35$ centimetres.

138. What is the true barometric height when the faulty barometer in the preceding example indicates 65 centimetres ?

139. When a barometer stands at 75 centimetres, the volume of the empty space above the mercury is 8 cubic centimetres, and its length is 13 centimetres : how far would the column fall if you introduced 5 cubic centimetres of air into the tube ?

140. A quantity of air is collected in a barometer tube, the mercury standing 10 inches higher inside the tube than outside, and the correct barometric height being 30 inches. If the volume of the air under these conditions is 1 cubic inch, what would be its volume at the atmospheric pressure ?

141. A barometer which has not been completely freed from air reads 76 centimetres when the correct barometric height is 77 centimetres, the free internal height of the top of the barometer tube above the surface of the mercury in the cistern being 85 centimetres: show that when this defective barometer reads 75 centimetres a true barometer would stand at 75·9 centimetres.

142. The tube of a barometer has a cross-section of 1 square centimetre, and when the mercury stands at 75 centimetres the volume of the vacuum above it is 12 cubic centimetres: what would be the volume at the same pressure (75 centimetres) of the air which, when introduced into this barometer, would depress the mercury (1) to 30 centimetres, and (2) to 0 centimetre?

143. A cylindrical diving-bell, 7 feet in height, is lowered until the top of the bell is 20 feet below the surface of the water. If the height of the mercury barometer at the time is 30 inches, how high will the water rise inside the bell?

Let x be the height of the space occupied by the air inside the bell; the height of the column of water exerting pressure upon this is $(20+x)$ feet. The pressure of the atmosphere is equal to that of a column of mercury 30 inches high, or of a column of water whose height is

$$30 \times 13{\cdot}6 = 408 \text{ in.} = 34 \text{ ft.}$$

(This may be expressed by saying that the height of the water barometer is 34 feet.)

The total pressure is equal to that of a column of water whose height is $34+20+x=(54+x)$ feet. Since the section of the bell is uniform, the original and final volumes are as 7 to x; and hence, applying Boyle's law,

$$34 \times 7 = (54+x)x,$$

or

$$x^2 + 54x - 238 = 0.$$

The positive value of x is 4.1; thus the water rises 2·9 feet within the bell.

Note.—In Examples 144-150 the height of the mercury barometer may be taken as 30 inches and that of the water barometer as 34 feet.

144. A cylindrical diving-bell, 9 feet high, is immersed in water so that its top is 27 feet below the surface: find how high the water rises within it.

145. A cylindrical diving-bell, 8 feet in height, is lowered to the bed of a river 48 feet deep: to what height will the water rise within the bell?

146. To what depth must a diving-bell 6 feet high be immersed so that the water may rise 4 feet within it?

147. If the cross-section of the diving-bell in Example 143 was 14 square feet, how much air at the atmospheric pressure would have to be introduced in order to keep the water from entering?

The bell originally contained $7 \times 14 = 98$ cubic feet of air, under a pressure $= 34$ feet of water. When the bell is filled with air and its top is 20 feet below the surface, the total pressure on the air is $34 + 20 + 7 = 61$ feet of water. Hence, by Boyle's law, if v cubic feet of air at atmospheric pressure have to be introduced,

$$34 (98 + v) = 61 \times 98,$$

and

$$\therefore v = 98 \times 27/34 = 77{\cdot}82 \text{ cub. ft.}$$

148. Find how much air would have to be pumped in so as to keep the water from entering if, in the preceding example, the bell had been lowered until the top was 50 feet below the surface of the water. What would be the height of a mercury barometer within the bell when the latter was filled with air?

149. An inverted test-tube, 6 inches long, and of uniform cross-section, is just immersed in mercury of specific gravity 13·6: how high does the liquid rise inside the test-tube?

150. A cylindrical gas-jar, 1 foot in length, is plunged mouth downwards in sea-water of density 1·026: to what depth must it be sunk in order that the water may rise half way up the jar?

151. The volume of the receiver of an air-pump is R, and that of the barrel is B: prove that if the density of the air in the receiver before exhaustion is D, the density after n complete strokes is,

$$D_n = \{R/(R+B)\}^n D.$$

152. The receiver of an air-pump has three times the volume of the barrel: find the density after ten complete strokes. (Use logarithms: see § 12.)

153. After three complete strokes the density of the air in the receiver of an air-pump was to its original density as 125 : 216: show that the volume of the receiver was five times that of the barrel.

154. The receiver of an air-pump has a volume of 1350 cubic centimetres, and it contains air of density 0·001293 at the normal pressure. The pump is now worked until the density of the air is reduced by 25 per cent: find the mass of air that has been removed.

155. Two bodies, whose volumes are 600 cubic centimetres and 30 cubic centimetres respectively, are suspended from the arms of a balance, and exactly balance each other in vacuo: find what weight must be attached to the large body in order that they may balance each other in air. (1 cubic centimetre of air = 0·0013 gramme.)

156. If w denote the weight of a body in vacuo, and w' its weight in water, prove that its weight in air of density δ will be

$$w - (w - w')\delta.$$

157. The weights in vacuo of two portions of the same substance are w and w'. The first portion is placed in the upper cup of a Nicholson's hydrometer in presence of air of density δ; and the hydrometer is immersed to the same depth as when the second portion is placed in the lower cup: show that the density of the substance is

$$(w' - w\delta)/(w' - w).$$

158. The volume of a balloon filled with coal-gas is

1,000,000 litres, and its mass (including the car) is 500 kilogrammes. If the density of the air is 1·28, and that of the coal-gas 0·52, both being in grammes per litre, find what additional weight the balloon can sustain in the air.

EXAMINATION QUESTIONS.

159. Define the measure of the elasticity of a fluid, and prove that if the elasticity is equal to the pressure, the pressure is inversely proportional to the volume.

M. Tripos. 1875.

160. If a centimetre cube of water be called a gramme, prove that the number of grammes in the earth is $6·15 \times 10^{27}$, the diameter of the earth being taken to be $1·275 \times 10^9$ centimetres, and the mean density to be 5·67. Camb. Schol. 1885.

161. A substance of specific gravity s when mixed with $1/n$ of its volume of A gives a mixture of specific gravity s_1, and when mixed with $1/m$ of its *weight* of B gives a mixture of specific gravity s_2: if it be mixed with $1/n$ of its volume of A and $1/m$ of its weight of B, what will be the specific gravity of the mixture?

Vict. B. Sc. 1884.

162. Equal volumes of three fluids are mixed, and the mixture separated into three parts: to each of these parts is then added its own volume of one of the original fluids, and the densities of the mixtures so formed are in the ratios of 3 : 4 : 5: prove that the densities of the fluids are as 1 : 2 : 3. Owens Coll. 1885.

163. A quantity of heavy liquid is at rest under the action of gravity, its surface extending over a considerable area. State what you know respecting (1) the form of the free surface; (2) the relation between the pressures in different directions at any point; (3) the magnitude of the pressure at different depths below the surface.

How might the value of "g" in different latitudes be determined by means of a delicate pressure-gauge immersed to a constant depth, say 20 feet, beneath the surface of still water? Matric. 1882.

164. Find the pressure in grammes weight per square centimetre at the bottom of a vessel one and three-quarter metres deep, full of mercury. (Specific gravity of mercury 13·596.) Glasgow M.A. 1882.

165. A cubical vessel with vertical sides, each 100 centimetres in area, is filled half with mercury and half with water: find the pressure in dynes on one of its sides, and the position of the point through which the resultant fluid pressure on this side acts. ($g=981$ centimetres, specific gravity of Hg = 13·6.) Balliol Coll. 1881

166. Distinguish between the whole pressure and the resultant pressure of a heavy fluid on a solid immersed in it. Find each, in pounds weight, for a cube of 1 foot edge with its centre 10 feet below the surface of still water, assuming a cubic foot of water to contain 1000 ounces. Prel. Sc. 1886.

167. A hole, 6 inches square, is made in a ship's bottom 20 feet below the water-line. What force must be exerted in order to keep the water out by holding a piece of wood against the hole, if a cubic foot of sea-water contains 64 pounds? Matric. 1886.

168. A cubical box filled with water is closed by a lid without weight which can turn freely about one edge of the cube; a string is tied symmetrically round the box in a plane which bisects the edge: show that if the lid be in a vertical plane with this edge uppermost, the tension of the string is one-third the weight of the water. Camb. Schol. 1886.

169. Two indefinitely long cylinders equal in all respects are placed on a horizontal plane, their bases being connected by a horizontal pipe of small section. The cylinders are filled to depths of 2 feet and 1 foot by fluids of densities 3ρ and 6ρ respectively. Show that if a volume of fluid of density 4ρ, which would occupy a length of either cylinder greater than 3 feet, be poured into the second cylinder, the free surfaces of the fluids in the cylinders will lie on the same horizontal plane.

Camb. Schol. 1883.

170. Show how the pressure of the atmosphere may be expressed in absolute units. Two vertical cylinders are filled with water to the same level; they are connected at the bottom by a horizontal cylindrical tube 1 centimetre in diameter and 1·5 metre below the level of the water; the tube is closed by a trap-door: find the force upon the trap-door in dynes if the temperature of the water on the one side be 15°C. and on the other be 16°C.

Density of water at 15°C. = ·999173 gm. per c.c.
" " 16°C. = ·999015 " "

Camb. Schol. 1886.

171. A piece of lead weighing 17 grammes, and a piece of sulphur, have equal apparent weights when suspended from the pans of a balance and immersed in water. When the water is replaced by alcohol of density 0·9, 1·4 gramme must be added to the pan from which the lead is suspended to restore equilibrium. Determine the weight of the sulphur, the density of the lead being 11·332.

Int. Sc. 1886.

This problem can be solved without knowing the density of the lead. For if V and V′ denote the volumes of the lead and sulphur, D and D′ their respective densities; then the equations of equilibrium reduce to

$$V(D-1)=V'(D'-1) \quad . \quad . \quad . \quad (1)$$

and

$$1{\cdot}4+V(D-0{\cdot}9)=V'(D'-0{\cdot}9).$$

Multiplying equation (1) by 0·9, we have

$$0{\cdot}9VD-0{\cdot}9V=0{\cdot}9V'D'-0{\cdot}9V',$$
$$\therefore\ 1{\cdot}4+0{\cdot}1VD=0{\cdot}1V'D'.$$

Now

$$VD=\text{weight of lead}=17\text{ gm.}$$
$$\therefore\ \text{weight of sulphur}=V'D'=31\text{ gm.}$$

172. A cube of glass of which the length of the edge is 3 centimetres is suspended in paraffin oil by a thread: find the apparent weight. (Specific gravity of glass 2·5, of paraffin oil 0·84.) Glasgow M.A. 1884.

173. A piece of silver and a piece of gold are suspended from the two ends of an equal armed balance beam, which is in equilibrium when the silver is immersed in alcohol (density = 0·85), and the gold in nitric acid (density = 1·5). The densities of the silver and gold being 10·5 and 19·3 respectively, what are their relative masses? Matric. 1885.

174. What is meant by the loss of weight of a body in a liquid?

A piece of lead and a piece of sulphur are suspended by fine strings from the extremities of a balance beam and just balance each other in water. Compare their volumes, their densities being respectively 11·4 and 2 grammes per cubic centimetre. Which of them will appear to be the lighter in air, and what weight must be added to it to restore equilibrium? Int. Sc. 1883.

175. Investigate the resultant pressure of a fluid on a body partially immersed in it.

Two solids whose weights are 4 pounds and $6\frac{1}{2}$ pounds, the volume of the former being double that of the latter, are connected by a weightless string passing over a smooth pulley, and rest in equilibrium totally immersed in fluids of specific gravity 1·3 and 3·24 respectively: find the volumes of the solids.

Cam. Gen. Exam. 1884.

176. A piece of wood floats partly immersed in water, and oil is poured on the water until the wood is completely covered. Explain clearly whether this will make any change (and if so, whether there will be an increase or a decrease) in the portion of the wood below the water. Matric. 1885.

177. What is meant by the specific gravity of a substance?

A body floats with one-tenth of its volume above the surface of pure water: what fraction of its volume would project above the surface if it were floating in liquid of specific gravity 1·25? Matric. 1883.

178. When a solid body floats in a certain liquid it is just half immersed. When it floats in a mixture of equal quantities of that liquid and water two-thirds of it are immersed: find the specific gravities of the solid and liquid. Edinb. M.A. 1885.

179. A piece of iron, weighing 275 grammes, floats in mercury of density 13·59 with five-ninths of its volume immersed: determine the volume and density of the iron. Matric. 1885.

180. Distinguish between the absolute and the apparent weight of a body. The apparent weight of a piece of platinum in water is 60 grammes, and the absolute weight of another piece of platinum twice as big as the former is 126 grammes: determine the specific gravity of platinum. Matric. 1886.

181. A specific gravity bottle full of water weighs 44 grammes, and when some pieces of iron, weighing 10 grammes in air, are introduced into the bottle and the bottle again filled up with water, the combined weight is 52·7 grammes: what is the specific gravity of the iron? Matric. 1887.

182. Find the specific gravity of a specimen of wood from the following data:—

Weight in air of specimen . . .	$28\frac{1}{2}$ gm.
„ air of brass sinker attached to specimen	25 „
„ water of specimen and sinker	1·9 „
Specific gravity of brass . . .	7·9 „

Glasgow M.A. 1882.

183. A cylindrical piece of cork, of height h, is floating with its axis vertical in a basin of water. If the basin be placed under the receiver of an air-pump, and the air be pumped out, prove that the cork will sink through the space $h\sigma (1 - s)/(1 - \sigma)$, where σ is the ratio of the densities of air and water, and s the ratio of the densities of cork and water. Ind. C. S. 1885.

184. Give a clear explanation of the statement that the mercury in a barometer tube is sustained by atmospheric pressure. What data would you require in order to calculate the height of a glycerine barometer when the pressure of the atmosphere is 15 pounds per square inch? Matric. 1885.

185. Glycerine rises in a barometer tube to a height of 26 feet when mercury stands at 30 inches. The specific gravity of mercury is 13·6: find that of glycerine. Matric. 1886.

186. A cylindrical tube a metre in length and having one end sealed contains dry air at the ordinary pressure and temperature. The tube is dipped vertically with its open end downwards into a tank of mercury, till the air within it is compressed to four-fifths of its former volume: find the distance of the top of the tube from the free surface of the mercury in the tank, the height of the barometer being 750 millimetres. Matric. 1879.

187. State the laws of compression for air and water respectively. Assuming them to hold for great ranges of pressure, show that a bubble of air will sink in water if the pressure be sufficiently increased; and calculate roughly at what pressure this will take place. (At a pressure of *one atmosphere* the density of air is 1/773 of the density of water.) Edinb. M.A. 1884.

188. If the height of the barometer at the sea-level be 760 millimetres, the specific gravity of mercury 13·6, and the specific gravity of sea-water 1·02, at what depth below the surface is the pressure per square centimetre equivalent to the weight of 10 kilogrammes?

Camb. Schol. 1883.

189. A barometer stands at 30 inches, and the space occupied by the Toricellian vacuum is then 2 inches; if now a bubble of air which would at atmospheric pressure occupy half an inch of the tube be introduced into the tube, prove that the surface of the mercury in the tube will be lowered 3 inches. Show also that the height of a correct barometer when this false one stands at x inches is $x + 15/(32 - x)$. Camb. Schol. 1885.

190. A cylindrical diving-bell of length $h/4$ is sunk in water till its lowest part is nh below the surface; if the water fill $\frac{1}{5}$ of the bell, show that the bell contains air whose volume at atmospheric pressure would be $\frac{4}{5}(n + \frac{19}{20})V$, V being the volume of the bell and h the height of the water barometer. Camb. B.A. 1880.

191. The height of the water barometer is $33\frac{1}{3}$ feet; a bubble of gas has a volume of 1 cubic inch at a depth of 100 feet below the surface of pure water: what will be its volume on reaching the surface? Matric. 1884.

192. A cubic foot of water weighs 1000 ounces. A cylindrical test-tube is held in a vertical position and immersed mouth downwards in water. When the middle of the tube is at a depth of 32·75 feet it is found that the water has risen half-way up the tube: find the atmospheric pressure in pounds weight per square inch.
Matric. 1882.

193. Having given that the density of the air is ·00129, the density of mercury 13·596, and the height of the barometer 75·9 centimetres, prove that if the unit of force be taken to be the weight of 800 kilogrammes, the numerical value of the pressure of the air will be almost exactly equal to that of its density, it being assumed that a centimetre cube of water weighs 1 gramme. M. Tripos. 1880.

194. The height of the water barometer being 33 feet 9 inches, and the specific gravity of mercury 13·5, find at what height a common barometer will stand in a cylindrical diving-bell when lowered until the water fills one-tenth of the bell. How far will the surface of the water within the bell be below the external surface?
Camb. Schol. 1886.

195. Suppose that a cubic foot of air weighs 1·2 ounce, and a cubic foot of water 1000 ounces. A balloon so thin that the volume of its substance may be neglected contains 1·5 cubic feet of coal-gas, and the envelope, together with the car and appendages, weighs 1 ounce. The balloon just floats in the middle of the room, without ascending or descending: find the specific gravity of coal gas (1) compared with air, (2) compared with water. Matric. 1882.

196. A cylindrical diving-bell weighs 2 tons, and has an internal capacity of 200 cubic feet, while the volume of the material composing it is 20 cubic feet. The bell is made to sink by weights attached to it. At what depth may the weights be removed, and the bell just not ascend, it being given that the mass of a cubic foot of water is 1000 ounces, and the height of the water barometer 33 feet? Matric. 1883.

197. What are the conditions of equilibrium of a floating body?

A Cartesian diver consists of a hollow ball with a weight attached to it. There is an opening at the bottom of the ball, and it contains a quantity of air. If the diver be made of glass (specific gravity = 3·33), and weigh 7·05 grammes, find the volume of air in it in order that it may just float in water at its maximum density, and show that if it contain 5·2 cubic centimetres of air and be sunk to a depth of 500 millimetres in the water it will not rise again, the pressure of the air above the water being that due to 10 metres of water. (The weight of the air in the diver may be neglected.)

Camb. Schol. 1881.

198. Atmospheric air having a volume of 50 litres at ordinary pressure (1000 grammes weight per square centimetre) is admitted into a vessel having a volume of ·1 cubic metre, and containing only aqueous vapour at a pressure of 4 grammes weight per square centimetre. What is the pressure of the mixture in grammes weight per square centimetre, and in centimetres of mercury? (Density of mercury = 13·59 grammes per cubic centimetre. The temperature is supposed to remain constant throughout.) Prel. Sc. 1887.

199. Explain the action of a syphon. A syphon tube with vertical arms filled with mercury (ρ) and closed at both ends is inserted into a basin of water (σ). When the stoppers are removed examine what will ensue, and prove the following results if the barometer is sufficiently high:

(i.) If k, the whole length of the outside arm, exceeds h, the whole length of the immersed arm, the mercury will flow outwards and the water will follow it.

(ii.) If $h > k$, the ends of the immersed tube must be at a depth below the free surface of the water exceeding $(h - k)\,\rho/\sigma$ in order that the mercury may not flow back into the basin. M. Tripos. 1885.

CHAPTER III

HEAT—EXPANSION

1. Expansion of Solids

Definition.—The coefficient of linear expansion of a substance is the increase in length produced in unit length of the substance by a rise in temperature of 1 degree centigrade.

APPROXIMATE COEFFICIENTS OF LINEAR EXPANSION.

Glass	0·0000086
Platinum	0·0000086
Iron	0·000012
Copper	0·000017
Brass	0·000019
Zinc	0·000029

These values may be assumed in solving the examples in this chapter. All temperatures are expressed on the centigrade scale.

Let α denote the coefficient of expansion of a body whose length at 0° is l_0: on heating to $t°$ the expansion produced will be $l_0\alpha t$. The length l_t of the body at $t°$ will be given by the equation—

$$l_t = l_0 + l_0\alpha t = l_0(1 + \alpha t) \quad . \quad . \quad . \quad (1)$$

1. Find the length at 200° of a zinc rod whose length at 0° is 128 cm.

If the length at 200° be denoted by l_{200}, then

$$\begin{aligned} l_{200} &= l_0(1 + 200\alpha) \\ &= 128(1 + 200 \times 0{\cdot}000029) \\ &= 128 \times 1{\cdot}0058 \\ &= 128{\cdot}7424 \text{ cm.} \end{aligned}$$

2. A piece of brass wire is exactly 3 metres long at 250°: what will be its length at 0°?

Using the same system of notation, we have, by equation (1),

$$\begin{aligned} l_0 &= l_{250}/(1+250a) \\ &= 300/(1+250\times 0{\cdot}000019) \\ &= 300/1{\cdot}00475 = 298{\cdot}582 \text{ cm.} \end{aligned}$$

If the length l_t of a metal bar at $t°$ is given, and its length $l_{t'}$ at another temperature t' is required, we may find its length at 0° as in the preceding example, and then proceed as in Ex. 1. But a result which is very nearly correct may be obtained more directly by means of the equation

$$l_{t'} = l_t(1 + a\cdot\overline{t'-t}) \quad . \quad . \quad . \quad (2)$$

This formula may be proved as follows. By equation (1) we have

$$l_t = l_0(1+at),$$

and

$$l_{t'} = l_0(1+at'),$$

$$\therefore\ l_{t'} = l_t(1+at')/(1+at).$$

Now the coefficients of expansion of metals are expressed by numbers which are very small compared with unity; and unless t and t' are very high temperatures, at and at' will also be small compared with unity. It therefore follows that $(1+at')/(1+at) = 1 + at' - at$ (see § 11), and

$$l_{t'} = l_0(1 + a\cdot\overline{t'-t}), \text{ approximately.}$$

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3. A steel metre-scale was carefully measured at 10° and its length was found to be 99·981 cm. At 40° its length was found to be 100·015 cm.: calculate the coefficient of expansion of the steel, and find the temperature at which the scale is exactly 1 metre long.

By equation (2) we have

$$\begin{aligned} a &= (l_{t'} - l_t)/l_t(t'-t) \\ &= (100{\cdot}015 - 99{\cdot}981)/99{\cdot}981 \times 30 \\ &= 0{\cdot}034/2999{\cdot}43 = 0{\cdot}0000113. \end{aligned}$$

If x° be the temperature at which the scale is correct,

$$100 = 99{\cdot}981\,\{1 + 0{\cdot}0000113(x - 10)\},$$
$$\therefore x - 10 = 0{\cdot}019/0{\cdot}0000113 \times 99{\cdot}981,$$
and $$x = 10 + 16{\cdot}82 = 26^\circ{\cdot}82.$$

4. A copper rod, the length of which at 0° is 2 metres, is heated to 200°: what will its length now be? At what temperature will its length be 200·51 centimetres?

5. The length of a glass tube at 100° is 154 cm.: what would its length at 0° be?

6. An iron yard-measure is correct at the temperature of melting ice: express, as a fraction of an inch, its error at the temperature of boiling water.

7. What is the length of a brass wire which on heating through 200° increases in length by 1 centimetre?

8. The distance between two points appears to be 87·2 cm., when measured at 28° on a brass scale which is right at 0°: what is the real distance?

9. What must be the length at 50° of a brass standard yard-measure, in order that it may be exactly correct at the freezing-point?

10. The length of the iron railway bridge across the Menai Straits is about 461 metres: find the total expansion of this iron tube between -5° and 35°.

11. A rod which is exactly $2\frac{1}{4}$ metres long at 10° is heated to 160°, when its length is found to be 2·277 metres: what is its coefficient of expansion? At what temperature will its length be 2·295 metres?

12. A copper wire is found to be 0·034 cm. longer at 25° than it is at 5°. Calculate accurately what its length at 0° would be.

13. A metal scale was examined at 32° F. and 76° F., and was found to have expanded 0·014 inch between these temperatures: find what change would be produced in its length by an elevation of temperature of 50° C.

14. Calculate the superficial expansion produced by a rise of 40° C. in a plate of sheet-iron 5 ft. long and 3 ft. broad.

15. A steam-pipe, intended to convey steam at 110°, is formed of iron piping in lengths of 15 ft.; assuming that the temperature of the pipe, when it is not conveying steam, is 12°, find how much play must be allowed at each joint.

16. Two long slips of metal, one of iron, the other of brass, are firmly riveted together: describe what will happen when they are heated, and explain how the unequal expansion of metals has been applied:—

(1) In the determination of temperature.

(2) In the construction of delicate chronometers.

17. A sheet of brass is 20 cm. long and 15 cm. broad at 0°: what is its superficial area at 80°?

18. A rod of iron and a rod of brass are placed side by side, and firmly riveted together at one end. Their lengths at 0° are 1·5 and 2·5 metres respectively, so that the distance between their free ends at this temperature is exactly 1 metre. The compound bar is immersed in a hot oil-bath at a temperature of 220°: what is now the distance between the free ends?

19. Show that the lengths of the metal bars of a compensation-pendulum should be inversely proportional to the coefficients of expansion of the metals. If the length of the iron bars be 87 centimetres, what should be the length of the zinc bars?

20. The time of vibration of a pendulum is proportional to the square root of its length, and a certain clock with an iron pendulum rod is made so as to keep correct time at 5°: how will its rate alter if the temperature rises to 30°?

21. A clock which keeps correct time at 25° has a pendulum rod made of brass: how many seconds a day will it gain if the temperature falls to the freezing-point?

22. A spherical iron ball, of 5·01 cm. diameter at 0°, rests upon a copper ring, the internal diameter of which is exactly 5 cm. at the same temperature. To what temperature must both be heated in order that the ball may just pass through the ring?

Let t be the required temperature. The diameter of the iron ball at $t°$ is $5{\cdot}01(1+0{\cdot}000012t)$, and the internal diameter of the copper ring at the same temperature is $5(1+0{\cdot}000017t)$. Assuming that the ball will just pass through the ring when their diameters are equal, we have

$$5{\cdot}01(1+0{\cdot}000012t)=5(1+0{\cdot}000017t),$$
$$\therefore 0{\cdot}01+(5{\cdot}01\times 0{\cdot}000012t)=(5\times 0{\cdot}000017t),$$

and

$$t=0{\cdot}01/(5\times 0{\cdot}000017-5{\cdot}01\times 0{\cdot}000012)$$
$$=0{\cdot}01/0{\cdot}00002488,$$

or $$t=402°.$$

23. A platinum wire and a strip of zinc are both measured at 0°, and their lengths are found to be 251 and 250 cm. respectively: at what temperature will their lengths be equal, and what will be their common length at this temperature?

24. Prove that the coefficient of contraction is, for most solids, approximately equal to the coefficient of expansion.

A glass tube which is 99·994 cm. long at 5°, and a brass rod of which the length at 22° is 100·019 cm., are found to be of exactly the same length at an intermediate temperature: what is this temperature?

25. Prove that the coefficient of cubical expansion of a substance is approximately three times its coefficient of linear expansion.

The coefficient of linear expansion of vulcanite is 0·00008: what change will be produced in the volume of a slab of vulcanite on heating it to 90°, if the slab at 0° is 1 ft. long, 10 in. broad, and 1 in. thick?

26. The volume of a leaden bullet at 0° is 2·5 cubic centimetres, and its volume at 98° is found to be 2·521 cubic centimetres: prove that the coefficient of cubical expansion of lead is 0·0000857.

27. The density of standard silver at 0° is 10·31, and its coefficient of cubical expansion is 0·000058 : find its density at 150°.

28. The coefficient of cubical expansion of sulphur is 0·000223, and a certain piece of sulphur is found to displace 48 c.c. of water at 0° : what volume of water will it displace at 35° ?

2. Expansion of Liquids.

29. Distinguish between the real and apparent expansion of a liquid contained in a glass vessel. If the apparent coefficient of expansion of mercury contained in a glass vessel is 1/6500, while its coefficient of absolute expansion is 1/5500, what is the coefficient of cubical expansion of the glass ?[1]

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[1] The absolute dilatation of a liquid is approximately equal to its apparent dilatation, together with the cubical expansion of the containing vessel. This may be proved as follows:—

Let V_0 be the common volume of the vessel and the liquid at 0°, V the real volume of the liquid at any temperature t, and V' its apparent volume at the same temperature.

If δ be the coefficient of absolute expansion of the liquid, its real volume at t is $V=V_0(1+\delta t)$; or, substituting Δ for δt,

$$V=V_0(1+\Delta) \quad . \quad . \quad . \quad . \quad \text{(a)}$$

The apparent volume of the liquid is

$$V'=V_0(1+D) \quad . \quad . \quad . \quad . \quad \text{(b)}$$

D being equal to dt, where d is the coefficient of apparent expansion.

V' is also the *apparent* volume of the portion of the vessel in which the liquid is contained at the temperature t. If k be the coefficient of cubical expansion of the vessel, its real volume is $V'(1+kt)$, and this is equal to V, the real volume of the liquid. Putting $kt=K$, we have

$$V=V'(1+K) \quad . \quad . \quad . \quad . \quad \text{(c)}$$

From equations (b) and (c), by multiplication, we have

$$V=V_0(1+D)(1+K).$$

30. Describe the behaviour of water when it is heated from 0° to 10° C. How would you show the existence of a point of maximum density between these temperatures?

31. The density of a liquid at 0° is D_0, and its coefficient of cubical expansion is k; show that its density at $t°$ is

$$D_t = D_0/(1 + kt).$$

32. Two thermometers, one filled with mercury, the other with water, are marked so as to agree at two fixed points, but their readings are found to differ at intermediate temperatures. Explain this fact, and discuss its bearing upon the general question of thermometric measurements.

Explain precisely what you understand by the symbol "200° C."

33. The volume of a gramme of water being 1 c.c. at 4°, and 1·0169 c.c. at 60°, what is the mean coefficient of expansion of water between these temperatures?

34. The density of water at 4° is unity, and its density at 60° is 0·9834: find its mean coefficient of expansion between 4° and 60°.

35. Describe Dulong and Petit's method of measuring the real expansion of mercury, giving a sketch of the apparatus employed. State the hydrostatic principle upon which the method is based, and show that if H_t and H_0 denote the height of the hot and cold columns respectively, the coefficient of expansion is

$$k = \frac{H_t - H_0}{H_0 \cdot t}.$$

Comparing this with equation (a), we see that

$$\begin{aligned} 1 + \Delta &= (1 + D)(1 + K) \\ &= 1 + D + K + DK. \end{aligned}$$

Now both D and K are small quantities compared with unity, and hence their product may be neglected (see § 11).

Therefore $\Delta = D + K$.

36. Point out the defects in the above method which render it difficult to make accurate measurements. Sketch and describe the modified form of apparatus introduced by Regnault.

37. In an experiment made by the above method, the heights of the mercury columns were 60 cm. and 61·09 cm., their temperatures being 0° and 100° respectively: what value does this give for the coefficient of expansion?

Note.—In Examples 38-47 the coefficient of real expansion of mercury is to be taken as 0·000182.

38. In an experiment made according to Dulong and Petit's method, the heights of the two columns of mercury were 90 cm. and 91·7 cm.: if the first column was at 0°, what was the temperature of the second?

39. Assuming that the density of mercury at 0° is 13·6, prove that its density at 120° is 13·3.

40. Show that the volume of a gramme of mercury at 110° is 0·75 c.c., and that the weight of 1 c.c. of mercury at 80° is 13·4 gm.

41. The height of the barometer is found to be 77.25 cm., the temperature of the air being 25°: prove that the corresponding barometric height reduced to zero would be 77·17 c.m., *i.e.* that the barometric column would stand at this height if the mercury were at 0°.

42. A specific gravity bottle contains exactly 687 gm. of mercury at 70°: show that its internal volume at this temperature is 51·171 c.c.

43. Prove that the mean coefficient of real expansion of mercury between 0° and 300° is 0·0001864, having given that its density is 13.595 at 0°, and 12·875 at 300°.

44. A weight thermometer contains M grammes of a liquid at 0°; on heating to a temperature t, m grammes of the liquid are expelled: show that the coefficient of apparent expansion of the liquid in its envelope is

$$k = m/(\mathrm{M} - m)t.$$

45. A glass bulb, with bent capillary tube, was filled with mercury at the temperature of melting ice: it was then heated to 100°, and the expelled mercury carefully collected and weighed. Calculate the apparent coefficient of expansion of mercury between these temperatures from the data given below:—

Weight of mercury expelled = 10·2877 gm.
„ of bulb + mercury at 100° = 697·5 gm.
„ of bulb alone = 38·5 gm.

46. A weight thermometer weighs 40 gm. when empty, and 490 gm. when filled with mercury at 0°; on heating it to 100°, 6·85 gm. of mercury escapes. Calculate the coefficient of expansion of the glass. (See notes on pp. 92 and 94.)

47. A glass flask contains 1·36 kilogramme of mercury at 0°: find the volume at 100° of the mercury which is expelled when the flask and its contents are heated to 100°. [Coefficient of cubical expansion of glass = 0·000025.]

48. A weight thermometer which contains a kilogramme of mercury at 0° is placed in an oil-bath, and the mercury expelled is found to weigh 20 grammes. Find the temperature of the bath, the coefficient of apparent expansion of mercury in glass being 0·000155.

49. A glass vessel with a capillary stem weighs 104·53 gm. when empty, and holds 623·51 gm. of mercury at 0°. What is the temperature when the whole apparatus weighs 717·62 gm.?

50. How much mercury would have been expelled if the glass vessel in the preceding example had been heated to 100° exactly?

51. A piece of glass, the weight of which in air was 46·76 gm., was found to weigh 31·29 gm. in water at its point of maximum density (4°), and 31·51 gm. in water at 60°. Find the coefficient of cubical expansion of water, taking that of glass as 0·000024.

The loss of weight of the glass when weighed in water at 4° is $46{\cdot}76 - 31{\cdot}29 = 15{\cdot}47$ gm., so that the volume of the glass at 4° is 15·47 c.c. At 60° its volume is

$$15{\cdot}47 \times (1 + 0{\cdot}000024 \times 56) = 15{\cdot}47 \times 1{\cdot}001344$$
$$= 15{\cdot}4908 \text{ c.c.}$$

Now the loss of weight on weighing in water at 60° is $46{\cdot}76 - 31{\cdot}51 = 15{\cdot}25$ gm.; therefore 15·4908 c.c. is the volume occupied by 15·25 gm. of water at 60°. This quantity of water at 4° would occupy 15·25 c.c., so that if k be the coefficient of expansion of water,

$$15{\cdot}4908 = 15{\cdot}25(1 + 56k),$$

and

$$\therefore k = \frac{0{\cdot}2408}{15{\cdot}25 \times 56} = 0{\cdot}000282.$$

52. The method indicated in the preceding example can also be employed for measuring the expansion of a solid when that of the liquid is known (Matthiesen's method). Explain fully how you would proceed to find *ab initio* the coefficients of cubical expansion of a solid and a liquid (say glass and water).

53. A glass rod which weighs 90 grammes in air is found to weigh 49·6 gm. in a certain liquid at 12°. At 97° its apparent weight in the same liquid is 51·9 gm.: find the coefficient of absolute expansion of the liquid.

The loss of weight at 12° is $90 - 49{\cdot}6 = 40{\cdot}4$ gm. If D be the density of the liquid at 12°, the volume of the glass at this temperature is

$$V = 40{\cdot}4/D.$$

The loss of weight at 97° is $90 - 51{\cdot}9 = 38{\cdot}1$ gm., and if D′ be the density of the liquid at this temperature, the volume of the glass at the same temperature is

$$V' = 38{\cdot}1/D'.$$

But since the coefficient of cubical expansion of glass is 0·000024,

$$V' = V(1 + \overline{85 \times 0{\cdot}000024})$$
$$= V \times 1{\cdot}00204.$$

Therefore

$$38{\cdot}1/D' = 1{\cdot}00204 \times 40{\cdot}4/D.$$

Again, if k be the coefficient of expansion of the liquid between 12° and 97°, $D' = D/(1+85k)$. Thus

$$38{\cdot}1 \times (1+85k) = 1{\cdot}00204 \times 40{\cdot}4,$$
$$= 40{\cdot}4824.$$
$$\therefore\ 38{\cdot}1 \times 85k = 40{\cdot}4824 - 38{\cdot}1 = 2{\cdot}3824,$$
and $$k = 2{\cdot}3824/38{\cdot}1 \times 85 = 0{\cdot}0007356.$$

54. A solid is found to weigh 29·9 gm. in a liquid of specific gravity 1·21 at 10°, its weight in air being 45·6 gm. It weighs 30·4 gm. in the same liquid at 95°, when its specific gravity is 1·17. Calculate the coefficient of cubical expansion of the solid.

3. Expansion of Gases.

Charles's Law.—The volume of a given mass of gas, kept at a constant pressure, increases by a definite fraction of its amount at 0° for each degree rise of temperature.

For air, hydrogen, oxygen and nitrogen, the value of this fraction (which is the coefficient of cubical expansion) is 0·00366, or approximately 1/273: the latter value may be adopted in solving the examples in this chapter.

Let V_0 denote the volume at 0° of a mass of gas whose coefficient of expansion is a; by Charles's Law, its volume at any other temperature $t°$ will be

$$\left.\begin{aligned} V_t &= V_0 + V_0 at = V_0(1+at), \\ &\text{or, since } a = 1/273, \\ V_t &= V_0\left(1+\frac{t}{273}\right) = V_0 \cdot \frac{273+t}{273} \end{aligned}\right\} \quad (3)$$

On the supposition that the law holds good down to − 273°, the volume of the mass of gas would appear to vanish at this temperature, which is called the absolute zero of temperature. Temperatures reckoned from this zero (– 273°) are called absolute temperatures.

If the volume V_t of a mass of gas be given, its volume V_t', at any other temperature t'° may be found either by first calculating its volume at 0°, or more simply as follows :—

By equation (3),

$$V_t = V_0(1 + at),$$

and

$$V_{t'} = V_0(1 + at').$$

Therefore

$$\frac{V_t'}{V_t} = \frac{1 + at'}{1 + at} \quad . \quad . \quad . \quad . \quad (4)$$

or, substituting for a its value 1/273,

$$\frac{V_t'}{V_t} = \frac{1 + t'/273}{1 + t/273} = \frac{273 + t'}{273 + t} \quad . \quad . \quad . \quad (5)$$

The volume of a given mass of gas, kept at a constant pressure, is therefore proportional to its absolute temperature.[1]

If we denote by T° and T′° the absolute temperatures corresponding to the temperatures $t°$ and $t'°$ in the centigrade scale, equation (5) reduces to the simpler form

$$\frac{V_{t'}}{V_t} = \frac{T'}{T} .$$

A gas is said to be at the normal pressure and temperature [N.P.T.] when its pressure is equal to that of a column of mercury 76 centimetres in height, and its temperature is 0° C. If a gas be heated while its volume is kept constant, the pressure increases just as the volume increases when the pressure is kept constant; so that if P_0 denote the pressure at 0°, the pressure at any other temperature $t°$ is

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[1] The student should compare this with what we have stated on p. 88 respecting the expansion of metal rods, and should be careful not to adopt the method there given when dealing with expansion of gases. The coefficients of expansion of gases are much larger fractions than those which express the coefficients of linear expansion of solids, and hence the approximate calculation on p. 88 is not applicable here.

$$P_t = P_0(1 + at) \quad . \quad . \quad . \quad . \quad (6)$$

a denoting here the coefficient of increase of pressure at constant volume, which has the same value as the coefficient of increase of volume at constant pressure (1/273). This last equation is similar to equation (3); and if we substitute P for V in the other equations (4 and 5), they will also hold good for change of pressure.

55. 100 c.c. of air is measured off at 20°: if the temperature be raised to 50°, what will the volume now be, the pressure remaining constant?

By equation (5), if V_{50} be the required volume at 50°,

$$\frac{V_{50}}{100} = \frac{273+50}{273+20} = \frac{323}{293},$$

and

$$\therefore V_{50} = 32300/293 = 110 \cdot 24 \text{ c.c.}$$

56. The weight of a litre of air at N.P.T. is 1·293 gm.: to what temperature must the air be heated so that it may weigh exactly 1 gm. per litre?

Let $t°$ be the required temperature, then at $t°$ 1·293 gm. of air occupies a volume of 1·293 litre, and the question is equivalent to the following: to what temperature must we heat a litre of air, taken at 0°, in order to increase its volume to 1·293 litre? The value of t is therefore given by the equation

$$1 \cdot 293 = 1 + t/273,$$

thus

$$t = 0 \cdot 293 \times 273 = 79 \cdot 99,$$

and the required temperature is 79°·99.

57. A closed flask containing air at 0° is connected with a mercury manometer which indicates that the pressure inside is less than that outside, the difference being equal to the pressure due to a column of mercury 15 cm. high. The flask is gradually heated: find the temperature at which the internal and external pressures will be equal, the barometric height being 75 cm. At what temperature will the pressure inside the flask be two atmospheres?

The pressure inside the flask is equal to that due to a column of mercury the height of which is $75 - 15 = 60$ cm. Suppose that at t° the pressure increases to 75 cm., then, by equation (6),

$$75 = 60(1 + t/273),$$
$$\therefore 60t/273 = 15,$$

and $$t = 273 \times 15/60 = 68^\circ{\cdot}25.$$

Again, if t'° be the temperature at which the pressure is 2 atmospheres ($= 150$ cm.),

$$150 = 60(1 + t'/273),$$
$$\therefore 60t'/273 = 90,$$

and $$t' = 273 \times 90/60 = 409^\circ{\cdot}5.$$

58. What will be the volume at 75° of a quantity of air which occupies 2·5 litres at 0°. At what temperature will its volume be exactly 3 litres?

59. At 50° the volume of a gramme of hydrogen is 13·2 litres: what is its volume (1) at 0°, (2) at 30°?

60. What volume of gas measured at 30° will have a volume of 200 c.c. at 0°?

61. A litre of hydrogen weighs 0·0896 gm. at 0°: find the weight of 1602 c.c. of hydrogen measured at 110°.

62. At what temperature will the volume of a given mass of gas be exactly double of what it is at 30°?

63. The stopcock of a copper boiler containing air at the normal pressure is closed when the temperature is 25°: it is then immersed, first in melting ice, and secondly in boiling water. Calculate the pressure (in centimetres of mercury) inside the boiler in both cases, neglecting the expansion of the boiler.

64. A certain mass of gas has a volume of 1250 c.c. at 90°: find its volume at 363°.

65. Calculate the mean coefficient of expansion of air between 0° and 200°, having given that its density is 0·001293 at 0° and 0·0007457 at 200°.

66. If the volume of a quantity of air at 10° be 230 c.c., at what temperature will its volume have increased to 285 c.c.?

67. An iron cylinder at 13° contains oxygen gas at a pressure of 6 atmospheres: if the cylinder is made to stand a pressure of 21 atmospheres, show that it may be heated to 728° before bursting.

68. Describe carefully an experimental method of finding the relation (1) between the volume and temperature of a gas kept at constant pressure, (2) between the pressure and temperature of a gas kept at constant volume.

69. A quantity of mercuric oxide is heated, and the oxygen given off is found to measure 300 c.c. On cooling to the temperature of the room (9°·5) the volume is reduced to 290·5 c.c.: what was the original temperature?

70. 6 litres of air at 10° are enclosed in the cylinder of an air engine, the cross-section of which is 200 sq. cm. The piston moves through a distance of 5 cm.: what elevation of temperature is required to keep the pressure constant?

71. A litre flask is filled with air at N.P.T.; the temperature is then raised to 130°, the pressure remaining constant: find the volume that would be occupied by the air which escapes, if it were again cooled to 0°. (You may neglect the expansion of the flask.)

72. The density of carbon monoxide is to that of carbon dioxide as 28 is to 44: prove that the density of carbon dioxide at 156° is equal to that of carbon monoxide at 0°, the pressures being identical.

Laws of Boyle and Charles.—Let v_0 be the volume at 0° of a mass of gas, the pressure being p_0. If the pressure changes to p (the temperature remaining constant), then, according to Boyle's Law, the volume becomes $v_0 \times p_0/p$. Now let the gas be heated to any temperature $t°$; according to Charles's Law, the volume will now become

$$v = v_0 p_0(1 + at)/p \quad . \quad . \quad . \quad (7)$$

and therefore

$$\left.\begin{aligned} \frac{pv}{1+at} &= p_0 v_0 \\ &= \text{a constant} = k \text{ (say)} \end{aligned}\right\} \quad . \quad . \quad (8)$$

Substituting for a its value $1/273$,

$$\frac{pv}{1+t/273} = k,$$

or

$$\frac{pv}{273+t} = \frac{k}{273} = \text{R (say)} \quad . \quad . \quad . \quad (9)$$

Since $273 + t = \text{T}$ (the absolute temperature corresponding to $t°$ on the centigrade scale), we may write the last equation in the simple form

$$\frac{pv}{\text{T}} = \text{const., or } pv = \text{RT} \quad . \quad . \quad . \quad (10)$$

Thus the product of the pressure and volume of a given mass of gas is always proportional to its absolute temperature.

73. A litre of dry air weighs 1·293 gm. at N.P.T. At what temperature will a litre of air weigh a gramme, the pressure being 72 cm.?

Let $t°$ be the required temperature. A litre of air, taken at 0°, would at $t°$ have the volume $(1 + t/273)$ litres, if the pressure remained constant; but when it changes from the normal pressure (76 cm.) to 72 cm., the volume increases further, and by the preceding proposition becomes

$$(1 + t/273) \times 76/72.$$

This is the volume (in litres) of 1·293 gm. of air at t°; and, in order that 1 gm. of air at this temperature should occupy exactly a litre, we must have

$$(1+t/273)\times 76/72\times 1\cdot 293=1.$$

Thus

$$(1+t/273)\times 76=72\times 1\cdot 293=93\cdot 1,$$
$$\therefore\ 76t/273=93\cdot 1-76=17\cdot 1,$$

and

$$t=273\times 17\cdot 1/76=61^\circ\cdot 43.$$

74. A quantity of air at the atmospheric pressure and at a temperature of 7° is compressed until its volume is reduced to one-seventh, the temperature rising 20° during the process: find the pressure at the end of the operation.

By the above proposition [equation (10)],

$$\frac{pv}{T}=\frac{p'v'}{T'},$$

the letters having the usual signification. The absolute temperatures corresponding to 7° and 27° are $T=273+7=280$, and $T'=273+27=300$. Taking the original volume as 7, and the final volume as 1, we have

$$1\times 7/280=p'\times 1/300$$
$$\therefore\ p'=7\times 30/28=7\cdot 5,$$

or the pressure at the end of the operation is $7\frac{1}{2}$ atmospheres.

75. A quantity of air is contained in a straight vertical tube closed at the lower end, the air being shut off by a pellet of mercury, the weight of which may be neglected. When the temperature is 13°, the mercury is 66 cm. from the bottom of the tube. What will be its position when the temperature is 52°, and what is the temperature when the mercury stands at 63 cm.?

Since the tube is supposed to be uniform, the volume occupied by the air is proportional to the distance of the mercury from the bottom of the tube. Let d be the distance when the temperature is 52°: by equation (5)

$$\frac{d}{66}=\frac{273+52}{273+13}=\frac{25}{22},$$

and $$d=3\times 25=75 \text{ cm.}$$

Again, if t be the temperature when the mercury stands at 63 cm., by the same equation,

$$\frac{63}{66}=\frac{273+t}{273+13},$$

$$\therefore\ 273+t=286\times 63/66=273,$$

or the required temperature is 0°.

76. A quantity of a certain gas was collected and found to measure 54·02 c.c. at a temperature of 22°, the barometric height being 74 cm. On cooling down to 0° the volume became 49·3 c.c., the height of the barometer at the time being 75 cm.: calculate the coefficient of expansion of the gas.

Let a denote the coefficient of expansion; then, since the volume of the gas at 0° and 75 cm. was 49·3 c.c., its volume at 0° and 74 cm. would, by Boyle's Law, be $49{\cdot}3\times 75/74=49{\cdot}97$ c.c., and at 22° and 74 cm. the volume would become $49{\cdot}97\times(1+22a)$.

Thus

$$49{\cdot}97\times(1+22a)=54{\cdot}02,$$

$$\therefore\ 49{\cdot}97\times 22\times a=54{\cdot}02-49{\cdot}97=4{\cdot}05,$$

and $$a=4{\cdot}05/49{\cdot}97\times 22=0{\cdot}003685.$$

77. Compare the volumes of equal masses of air (1) at the normal pressure and temperature, and (2) at 10° and 85 cm. pressure.

78. 450 c.c. of air is measured off at 0°: it is then heated to 30°, and the pressure is reduced to one-half: what is now the volume?

79. A quantity of air at atmospheric pressure is compressed so that its volume is reduced to one-tenth, the temperature being raised from 23° to 37°: express the new pressure in atmospheres.

80. Compare the masses of equal volumes of air when measured (1) at 0° and 30 in. pressure, and (2) at 65° and 29 in. pressure.

81. Compare the mass of 100 cubic feet of air at 40° and under the atmospheric pressure, with the mass of 10 cubic feet of air at 0° and under a pressure of 20 atmospheres.

82. If the barometer falls from 75 cm. to 70 cm., what must be the alteration in the temperature of a quantity of air originally at 17° in order that its volume may remain constant?

Note.—The density of dry air at N.P.T. is 0·001,293 gramme per cubic centimetre.

83. Show that the density of air at 76·8 cm. pressure and at 15° C. is 0·001239.

84. What is the mass of the air contained in a 500 c.c. flask at 10° and 73 cm. pressure?

85. The cubical content of a certain room is 750 cubic metres: calculate the mass of air contained in it at 17° and 77 cm. pressure.

86. The density of hydrogen is to that of air as 1 : 14·44: calculate the volume occupied by a gramme of hydrogen at 16° and 77 cm. pressure.

87. What is the weight of 10 litres of dry air at 14° and 74 cm. pressure?

88. The internal volume of a glass flask is 1 litre at 0°. It is filled with air at the normal pressure and temperature, and is then heated to 91° and opened under a pressure of 72 cm. Find the weight of the air that escapes.

89. Explain the construction of some practical form of air-thermometer, and the method of using it. What reasons have we for supposing that measurements of temperature made by such a thermometer are more reliable than those made by thermometers which depend upon liquid expansion?

90. If a mass of gas occupies a volume of 1750 c.c.

at 8° and 79 cm. pressure, what will be its volume at 26° and 74 cm. pressure?

91. A flask containing air is corked up at 20°: find the pressure inside the flask after it has stood for some time in a steam-bath at 98°, the original pressure being the standard atmospheric pressure of 30 inches of mercury.

If the flask can stand a pressure of $2\frac{1}{2}$ atmospheres, at what temperature will it burst?

92. Compare the densities of the air at the bottom and top of a mine-shaft, when the temperatures and barometric pressures are 20° C. and 31 inches, and 5° C. and 30 inches respectively.

93. A quantity of air is collected at 0° and 76 cm. pressure. The pressure now increases to 78 cm.: what change of temperature will cause the volume to increase up to its original value?

94. Compare the masses of air contained in a room (1) when the temperature is 6° and the barometer stands at 78 cm., and (2) when the temperature is 20° and the pressure 73 cm.

95. If the volume of a gas at 13° be doubled, to what temperature must it be raised in order that the pressure may not be affected by the change of volume?

96. A cylindrical test-tube 10 inches in length and containing air at 0° is inverted over a mercury-bath and forced downwards until its upper (closed) end is level with the surface of the mercury in the bath, the barometric height at the time being 30 inches: to what temperature must the bath be raised in order that the air may fill the test-tube?

97. A mixture is made of 8 litres of hydrogen at 74 cm. pressure and 3 litres of oxygen at 76 cm., both gases being at a temperature of 14°. The volume of the mixture is reduced to 10 litres: find the pressure, and prove that if the mixture is cooled to $-7°$ the pressure will be equal to the original pressure of the oxygen gas.

98. Equal quantities of air at temperatures t_1 and t_2 are contained in two hollow spheres whose radii are r_1 and r_2 respectively: prove that the pressures within the spheres are as $(1+\alpha t_1)/r_1^3 : (1+\alpha t_2)/r_2^3$, and that the whole pressures on the internal surfaces of the spheres are as $(1+\alpha t_1)/r_1 : (1+\alpha t_2)/r_2$.

99. A kilogramme weight is placed inside a bladder, which itself weighs 50 grammes. The bladder is then partly filled with air at 0° and tied up, when its volume is exactly 1 litre: find the temperature at which it will just float in water, the expansion of the water itself being neglected.

EXAMINATION QUESTIONS.

100. Explain what is meant by the coefficient of expansion of a substance.

The rod of a two-seconds pendulum compensated on the ordinary zinc and iron principle consists of an iron rod 13·4 ft. long, the lower part of which passes through a zinc tube which rests on a nut at the bottom of the rod. This tube is surrounded by another of iron of the same length as itself, which hangs from a cap attached to the top of the zinc tube. The centre of gravity of the bob is attached to the lower end of the iron tube. The coefficient of expansion of zinc is ·0000167 and that of iron ·0000067. Show that the length of the tubes should be 8·978 ft. Camb. B.A. 1878.

101. A tubular bridge made of wrought-iron has a surface area of 160,000 square feet. Show that it would cost £2 : 9s. more to paint it at 6d. per square foot when the temperature is 29° C. than when it is 4° C., the coefficient of linear expansion of wrought-iron being ·00001225. Camb. B.A. 1883.

102. A rod of copper and a rod of iron, placed side by side, are riveted together at one end. The iron rod is 150 cm. long, and a mark is made on the copper rod showing the position of the unriveted end of the iron at

0° C. If at 30° the mark is 0·0255 cm. from the end of the iron rod, what is the coefficient of expansion of copper, that of iron being 0·000012?

Univ. Coll. Lond. 1886.

103. Distinguish between the absolute and the apparent expansion of mercury contained in a thermometer.

The coefficient of absolute (cubic) expansion of mercury is ·00018, the coefficient of linear expansion of glass is ·000008. Mercury is placed in a graduated glass tube, and occupies 100 divisions of the tube. Through how many degrees must the temperature be raised to cause the mercury to occupy 101 divisions?

Matric. 1883.

104. How is the apparent related to the absolute expansion of a liquid?

A glass bottle holds 1359·6 grammes of mercury at the temperature of melting ice. If the temperature be raised to that of boiling water, how much mercury will be expelled from the bottle, the coefficient of apparent expansion of mercury in the glass being 0·000154?

Matric. 1886.

105. A glass bottle holds at 0° C. 10169·3 grains of mercury, while at 100° C. it only holds 10011·4 grains. Assuming that the dilatation of mercury between 0° and 100° is ·018153, find the coefficient of cubical expansion of the glass bottle. Owens Coll. 1886.

106. Describe some method by which the expansion of water has been studied.

If δ be the expansion of water between 4° and 0° C., and Δ its expansion between 4° and $t°$, show what is the density of water at $t°$ referred to water at 0°.

Int. Sc. 1884.

It should be noticed that the change of volume is *positive on either side of* 4°, because water expands on cooling from 4° downwards. Thus if unit volume of water be taken at 4°, its volume at 0° will be $(1 + \delta)$, and at $t°$ will be $(1 + \Delta)$. Let d_0 and d_t represent the densities of water at 0° and $t°$

respectively. Since the mass of water remains the same, the product of the volume into the density remains constant.

$$\therefore (1+\delta)d_0 = (1+\Delta)d_t,$$

or $$d_t = d_0(1+\delta)/(1+\Delta),$$

which is the required relation.

107. Given the absolute expansion of mercury between two fixed temperatures, show how to determine the absolute expansion of a liquid between these temperatures by means of a weight thermometer about which nothing is previously known.

Thermometers constructed with liquids A, B agree with a mercurial thermometer at temperatures $O°$ $T°$, $O°$ $T'°$ respectively. Find at what temperature on the mercurial thermometer they give the same reading, having given

Apparent expansion of $A = a_1t + a_2t^2$,

" " $B = b_1t + b_2t^2$,

where t is measured on the mercurial thermometer.

Oxford 1885.

108. A solid weighs 320 grammes in vacuo, 240 grammes in distilled water at 4° C, and 242 grammes in water at 100° C., of which the density is 0·959. Find the volume of the solid at these two temperatures, and deduce therefrom its mean coefficent of cubical expansion for 1° C. Int. Sc. 1876.

109. Describe carefully some method of measuring the coefficient of expansion by heat of a solid substance.

The apparent mass of a piece of glass weighed in water at 4° is 25 grammes, its real mass being 37·5 grammes; its apparent mass when weighed in water at 100° is 25·486 grammes. The coefficient of cubical expansion of glass per 1° C. is ·000026. Show that the volume of 1 gramme of water at 100° C. is 1·043 cubic centimetres. Ind. C. S. 1886.

110. Iron is 7·8 times as dense as water when they are compared at 4° C. What are their relative densities at 100° C.? The coefficient of linear expansion of iron is 0·000012, and the expansion of water between 4° and 100° is 0·043 of the volume at 4°. Prel. Sc. 1886.

111. The weight of a body in air is 10^k, in water at 4° is $9{\cdot}49998^k$, and in water at 10° is $9{\cdot}50007^k$: find the coefficient of expansion of the body and its density, given that 1 c.c. of water at 4° becomes 1·00025 c.c. at 10°. Balliol Coll. 1881.

112. A solid is weighed in a liquid at 0° C. and 100° C. The volume of the solid at 0° C. is unity and at 100° C. 1·006. Also the loss of weight by weighing in the liquid is, at 0° C., 1800 grains, and at 100° C., 1750 grains. Find the coefficient of dilatation of the liquid. Int. Sc. Honours 1877.

113. Suppose that the proportional cubical internal expansion of a glass sp. gr. bottle between 0° and 100° C. is ·00235, while the similar expansion of mercury is ·018153. Suppose also that, when the bottle contains a piece of iron weighing 2000 grains, the remainder of it will contain 6707·8 grains of mercury at 0° C., while at 100° C., under these circumstances, it will only contain 6599·4 grs. Finally, assume that the sp. gravities of mercury and iron at 100° C. are 13·2 and 7·8 respectively. Determine the cubical dilatation of iron between 0° and 100° C. B. Sc. 1877.

114. Experiments on the expansion of benzene gave the following results:—

Temperature.	Volume.
0°	1
20°	1·0241
40°	1·0500
60°	1·0776
80°	1·1070

Show that the coefficient of expansion can be represented by the formula $a+bt$, and determine the values of the constants a and b. B. Sc. 1884.

115. Determine the height of the barometer when a milligramme of air at 27° C. occupies a volume of 20 cub. cm. in a tube over mercury, the mercury standing 73 cm. higher inside the tube than outside. (1 gramme of air at 0° C. under a pressure of 76 cm. of mercury measures 773·4 cub. cm.) Int. Sc. 1885.

Let h denote the height of the barometer, then the pressure of the air in the tube over mercury is $h - 73$. At this pressure, and at 27°, a milligramme of air measures 20 c.c., whereas at 0° and 76 cm. pressure it measures 0·7734 c.c. By equation (9), p. 102, we have

$$\frac{(h-73)\times 20}{273+27}=\frac{76\times 0{\cdot}7734}{273},$$

$$\therefore h-73=76\times 0{\cdot}7734\times 300/20\times 273=3{\cdot}23,$$

and the required barometric height is 76·23 cm.

116. How may the relation between the pressure and temperature of a given mass of air at constant volume be determined?

A quantity of air occupies 10 cubic feet at 0° C. and under a pressure of 20 inches of mercury. What will be its volume at 30° C. and under a pressure of 1200 inches of mercury? Matric. 1884.

117. Explain accurately what is meant by the statement that the coefficient of expansion of air is 1/273. The volume of a certain quantity of air at 50° C. is 500 cubic inches. Assuming no change of pressure to take place, determine its volumes at 50° C. and at 100° C. respectively. Matric. 1887.

118. What is meant by saying that the absolute temperature of a gas is 300° C.? If the absolute temperature be 260° C., what is the temperature in the Centigrade scale?

Two condensers contain equal quantities of air. One of them at temperature 47° C. is 30 in. long, 20 in. broad, and 10 in. high; and the other at 57° C. is 30 in. long, 25 in. broad, and $8\frac{1}{4}$ in. high. Show that the pressure of the air is the same in both. Camb. B.A. 1883.

119. A vessel contains air at 0° C. and at atmospheric pressure. It is heated to 100° C., and during the process one ounce of air escapes. How many ounces of air were there originally in the vessel, the expansion of the vessel itself being neglected? The coefficient of expansion of air at constant volume is 1/273.

Matric. 1886.

120. A given quantity of a gas is made continually to occupy the same space. Explain what changes will take place in its pressure when changes take place in its temperature.

A straight vertical tube, the section of whose bore is one inch, is closed at its lower end, and contains a quantity of air which supports an air-tight piston whose weight is 1 lb. The position of the piston is observed when the temperature of the air is 31° C., and the weight of the piston is then increased by 1 lb. Find what increase of temperature will be required to bring back the piston to its former position, the atmospheric pressure being 15 lbs. per square inch, and the absolute zero of the air thermometer being $-273°$ C. Camb. B.A. 1884.

CHAPTER IV

SPECIFIC AND LATENT HEAT

1. Specific Heat.

1. A COIL of copper wire weighing 45·1 gm. was dropped into a calorimeter containing 52·5 gm. of water at 10°. The copper before immersion was at 99°·6, and the common temperature of copper and water after immersion was 16°·8. Find the specific heat of the copper wire.

The quantity of heat (Q) given out by a body of mass m and specific heat s in cooling through an interval of temperature θ is $Q = ms\theta$. Thus if s denote the specific heat of the copper, the amount of heat evolved by it in cooling from 99°·6 to 16°·8 is $45{\cdot}1 \times s \times (99{\cdot}6 - 16{\cdot}8)$.

Since the specific heat of the water is unity, the amount of heat required to raise its temperature from 10° to 16°·8 is $52{\cdot}5 \times 1 \times (16{\cdot}8 - 10)$, and as no heat is supposed to be gained or lost these two quantities are equal.

$$\therefore\ 45{\cdot}1 \times s \times 82{\cdot}8 = 52{\cdot}5 \times 6{\cdot}8,$$

and
$$s = 357/3734{\cdot}3 = 0{\cdot}0956.$$

2. What is the temperature of an iron ball weighing 5 lbs., which, when immersed in 8 lbs. of water at 13°, raises the temperature to 48°? The specific heat of iron is 0·112.

If the temperature of the ball before immersion was $t°$, the number of heat-units[1] which it gives out, in cooling to the final temperature of 48°, is $5 \times 0{\cdot}112 \times (t - 48)$.

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[1] Since specific heat is merely a number, or a numerical ratio between quantities of heat, it is independent of the unit of

Assuming that no heat is gained or lost during the experiment, this must be equal to the number of heat-units absorbed by the cold water, *i.e.* to $8 \times (48 - 13)$. Equating these two quantities, we have

$$5 \times 0{\cdot}112 \times (t - 48) = 8 \times (48 - 13),$$

$$\therefore\ 0{\cdot}56t = 8 \times 35 + (0{\cdot}56 \times 48),$$

$$= 280 + 26{\cdot}88 = 306{\cdot}88,$$

and $$t = 548°.$$

3. In order to determine the specific heat of silver, a piece of the metal weighing 10·205 gm. was heated to 101°·9 and dropped into a calorimeter containing 81·34 gm. of water, the temperature of which was raised from 11°·09 to 11°·71. The water equivalent of the calorimeter, agitator, and thermometer employed was 2·91 gm.: find the specific heat of the silver.

The heat evolved by the hot body is $10{\cdot}205 \times s \times (101{\cdot}9 - 11{\cdot}71)$, where s is the sp. heat of the silver. The heat is partly absorbed by the water and partly by the calorimeter, etc., and these are together equivalent to $(81{\cdot}34 + 2{\cdot}91)$ gm. $= 84{\cdot}25$ gm. Since these are raised from 11°·09 to 11°·71, the heat absorbed is $84{\cdot}25\ (11{\cdot}71 - 11{\cdot}09)$. Equating these quantities, we have

$$10{\cdot}205 \times s \times 90{\cdot}19 = 84{\cdot}25 \times 0{\cdot}62,$$

and $$\therefore\ s = 0{\cdot}05677.$$

mass (or weight) employed; and this is also true for latent heat. In the statement or solution of a problem it is a matter of indifference whether we take as our unit of mass the pound, the kilogramme, or the gramme, provided that we use this unit consistently, the corresponding units of heat in the three cases being the pound-degree (or amount of heat required to raise one pound of water through 1° C.), the kilogramme-degree, and the gramme-degree. The last, which is the C.G.S. unit, is sometimes called a "calorie," and Berthelot distinguishes the kilogramme-degree from this by calling it a "Calorie" (1 Calorie = 1000 calories).

4. The same piece of silver was heated to 102°·2 and immersed in 75·3 gm. of turpentine at 10°·98: the experiment was performed with the same apparatus as in Ex. 3, and the final temperature was 12°·47. Calculate the specific heat of the turpentine.

Taking the sp. heat of the silver as 0·05677, the amount of heat which it gives out is

$$10{\cdot}205 \times 0{\cdot}05677 \times (102{\cdot}2 - 12{\cdot}47) = 51{\cdot}97.$$

Of this, $75{\cdot}3 \times s \times (12{\cdot}47 - 10{\cdot}98)$ is absorbed by the turpentine, s being its sp. heat; and the calorimeter absorbs

$$2{\cdot}91 \times (12{\cdot}47 - 10{\cdot}98) = 4{\cdot}336.$$

Equating the amounts of heat absorbed and emitted,

$$51{\cdot}97 = (75{\cdot}3 \times s \times 1{\cdot}49) + 4{\cdot}336,$$

and
$$s = \frac{47{\cdot}634}{75{\cdot}3 \times 1{\cdot}49} = 0{\cdot}425.$$

5. A certain vessel holds 800 c.c. of water at its temperature of maximum density (4°). How much heat must be imparted to the water before it begins to boil?

6. Define specific heat. A body of mass M and specific heat S at a temperature T° is dropped into a mass m of a liquid of specific heat s at $t°$: prove that the final temperature is

$$\theta = \frac{MST + mst}{MS + ms},$$

and that, if the liquid is water,

$$S = \frac{m(\theta - t)}{M(T - \theta)}.$$

7. How many units of heat are required to raise the temperature of 150 grammes of copper (of specific heat 0·095) from 10° to 150°?

8. What amount of heat must be given to an iron armour-plate 2 metres long, 1 metre broad, and 20 cm. in thickness, in order to heat it from 10° to 140°? [Sp. gr. of iron, 7·7; sp. heat, 0·112.]

9. The thermal capacity (or water equivalent) of a body being defined as the product of its mass into its specific heat, calculate the capacity for heat of a copper calorimeter of 125 grammes. What special name is given to the thermal capacity of unit mass of a substance?

10. A body of mass M at a temperature T° is dropped into a mass m of a liquid of specific heat s contained in a calorimeter of mass m' made of a substance of specific heat s', both the calorimeter and the liquid being at t°: if the final temperature is θ, prove that the specific heat of the solid is

$$S = \frac{(ms + m's')(\theta - t)}{M(T - \theta)}.$$

[Expressions such as those developed in Examples 6 and 10 are convenient when a number of similar problems have to be solved, or in calculating the results of actual laboratory examples where corrections have to be applied; but the student will find that in general it is best to work out problems in specific and latent heat directly by equating the quantities of heat evolved by the hot body and absorbed by the cold body, as in the solved examples 1-4.]

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11. Find the specific heat of a substance 100 grammes of which at 90°, when immersed in 250 grammes of water at 12°, gave a resulting temperature of 18°.

12. What is meant by the statement that the specific heat of water is thirty times the specific heat of mercury?

If a kilogramme of mercury at 120° is poured into a vessel containing 200 gm. of ice-cold water, what will be the temperature after the whole is mixed? How would the weight and material of the vessel affect the result?

Note.—In Examples 13-16 the specific heat of mercury is to be taken as 1/30.

13. Two pounds of boiling water are poured upon ten pounds of mercury at 16°: what will be the common temperature after mixing?

14. Compare the thermal capacities of equal volumes of water and mercury, the density of mercury being 13·6.

15. A flask containing half a litre of mercury at 0° is immersed in boiling water, and allowed to remain there until the mercury has attained the temperature of the water: how many heat-units does it gain from the water?

16. Equal masses of boiling water and of mercury at − 5° are mixed together: prove that the resulting temperature is 96°·65.

17. A kilogramme of mercury contained in a glass flask is heated by immersing the flask in a beaker of boiling water; it is then poured into a large flask containing 500 c.c. of water at 10°, and after shaking thoroughly the temperature is found to be 15°·3: find the specific heat of mercury.

What errors would probably occur in carrying out these operations, and how would they influence the result?

18. 35 grammes of copper are heated to 98°·5 and mixed with 30 grammes of water at 10°·3. The temperature of the mixture is found to be 19°·2: what is the specific heat of copper?

19. 61½ ounces of water are mixed with 100 ounces of alcohol, and the final temperature is found to be midway between the two initial temperatures: what is the specific heat of alcohol?

20. A pound of boiling water is allowed to cool down to 10°: if all the heat given out were employed in warming 40 pounds of air, initially at 0°, to what temperature would it be raised? [Sp. heat of air = 0·237.]

21. Calculate the specific heat of silver from the following data:—

Weight of silver	10·2 gm.
Weight of water	84·0 ,,
Temperature of silver . . .	102°
Initial temperature of water . .	11°·08
Final temperature . . .	11°·69

22. 30 grammes of iron nails at 100° are dropped into 60 grammes of water at 13°·2, and the final temperature is 18°·6. What is the specific heat of the nails?

23. If you had at your command a supply of boiling water and of tap-water at 10°, what quantities of each would you take in order to prepare a bath containing 20 gallons of water at 35°?

24. A 7-lb. iron weight was taken out of an oil-bath and immediately immersed in 10 lbs. of water at 8°, whereupon the temperature rose to 20°. If the specific heat of iron is 0·112, what was the temperature of the oil-bath?

This suggests a method of measuring high temperatures, such as those of furnaces: how would you carry it out practically?

25. Equal volumes of turpentine at 70° and of alcohol at 10° are mixed together: find the resulting temperature.

[Sp. gr. of turpentine = 0·87, of alcohol = 0·80.
Sp. heat of turpentine = 0·47, of alcohol = 0·62.]

Let v be the volume taken, and θ the resulting temperature. The mass of the turpentine is $0{\cdot}87v$, and the amount of heat which it evolves in cooling from 70° to θ° is $v \times 0{\cdot}87 \times 0{\cdot}47(70 - \theta)$. This is entirely spent in warming a mass $0{\cdot}8v$ of alcohol from 10° to θ°, for which operation $v \times 0{\cdot}8 \times 0{\cdot}62\,(\theta - 10)$ heat-units are required.

Equating these quantities, we have

$$0{\cdot}87 \times 0{\cdot}47(70 - \theta) = 0{\cdot}8 \times 0{\cdot}62(\theta - 10).$$

Thus

$$28{\cdot}623 + 4{\cdot}96 = \theta \times (0{\cdot}496 + 0{\cdot}4089),$$

and

$$\theta = 33{\cdot}583/0{\cdot}9049,$$
$$= 37^\circ{\cdot}11.$$

26. The densities of two substances are as 2 to 3, and their specific heats are 0·12 and 0·09 respectively: compare their thermal capacities per unit volume.

27. Assuming that the density of boiling water is 0·96, and that the density of mercury at 0° is 13·6, calculate the resulting temperature when equal volumes of boiling water and mercury at 0° are mixed.

28. The specific heat of air at constant pressure is 0·237, and a litre of air weighs 1·293 gramme: how much heat is given out by 50 litres of air in cooling from 25° to 5°?

29. Hot air at 650° is used for superheating steam which is originally at 100°. The air and steam are kept at constant pressure during the operation, under which circumstances their specific heats are 0·237 and 0·48 respectively, and they are introduced into the superheater in the proportion of 2 lbs. of air to 7 lbs. of steam. If the air is allowed to cool to 400°, to what temperature will the steam be raised?

30. Three liquids, A, B, and C, are at temperatures of 30°, 20°, and 10° respectively. When equal parts (by weight) of A and B are mixed, the temperature of the mixture is 26°; and when equal parts by weight of A and C are mixed, the temperature is 25°. Prove that a mixture of equal parts of B and C will have a temperature of $16^\circ\frac{2}{3}$.

Let S_a, S_b, and S_c denote the specific heats of the liquids A, B, and C respectively. The equation for the amounts of heat evolved and absorbed when equal parts of A and B are mixed reduces to

$$(30-26)S_a=(26-20)S_b,$$

and

$$\therefore S_b=\frac{2}{3}S_a.$$

Similarly, when A and C are mixed,

$$(30-25)S_a=(25-10)S_b,$$

and $$\therefore\ S_c = \frac{1}{3} S_a.$$

If θ be the temperature of a mixture of equal parts of B and C,

$$(20 - \theta)S_b = (\theta - 10)S_c,$$

i.e. $$(20 - \theta) \cdot \frac{2}{3} \cdot S_a = (\theta - 10) \cdot \frac{1}{3} \cdot S_a,$$

$$\therefore\ 40 - 2\theta = \theta - 10, \text{ and } \theta = \frac{50}{3} = 16^\circ \frac{2}{3}.$$

31. 10 grammes of a liquid at 90° were mixed with an unknown quantity of a second liquid of specific heat 0·25 and temperature 16°; the resulting temperature was 43°·75. If the specific heat of the first liquid was 0·45, what was the weight of the second?

32. A liquid of specific heat 0·54 and temperature 29° is mixed with another liquid of specific heat 0·36 and temperature 11°, and the final temperature was 17°. In what proportions were the liquids mixed?

33. Equal weights of three liquids, whose specific heats are s_1, s_2, and s_3, and temperatures t_1°, t_2°, and t_3° respectively, are thoroughly mixed: find the temperature of the mixture.

34. In order to find the specific heat of absolute alcohol, a quantity of it was boiled in a test-tube, and poured at its boiling-point (78°·5) into a calorimeter containing 74 gm. of turpentine at 10°·6. The calorimeter was weighed before and after the addition; the gain of weight was 14·7 gm., and the final temperature was 25°·2. Find the specific heat of the alcohol, that of the turpentine being 0·466.

What advantage was there in using turpentine in this experiment?

35. 200 c.c. of water at 55° is poured into a copper calorimeter whose mass is 30 gm. and specific heat 0·095. Assuming that the calorimeter was previously at the temperature of the air, viz. 10°, and that the whole of the heat evolved by the water in cooling is absorbed by the copper, find the temperature to which the water is cooled.

36. 10 grammes of a metal were taken for a specific heat determination, and by a preliminary experiment it was found that the rise of temperature was insufficient, being only 3°·7. The quantity of water in the calorimeter was reduced by one-third: what weight of the metal should now be taken to produce a rise of 12°, the other conditions remaining the same?

37. 40 gm. of water at 45° were poured into a leaden crucible weighing 300 grammes, which had previously been standing in a room the temperature of which was 16°. The water was cooled to 39°·46: what is the specific heat of lead?

38. In two experiments made to determine the sp. heat of lead shot, the water equivalent of the calorimeter was 1·3, and that of the thermometer was 0·5. You are required to find the mean value from the results given, allowing for the heat absorbed by the calorimeter and thermometer.

	Exp. I.	Exp. II.
Weight of water . .	48·1 gm.	52·4 gm.
Weight of shot . .	60·9 ,,	90·0 ,,
Temperature of shot .	100°	100°
Initial temp. of water .	13°·0	14°·15
Final ,, ,, .	16°·2	18°·5

39. Describe the method employed by Regnault in determining the specific heats of gases at constant pressure, explaining in detail the construction of the heating apparatus and calorimeter, and the means adopted for obtaining a current of the gas at an uniform pressure.

40. If the quantity of heat required to raise unit mass of a substance from 0° to t° be represented by

$$Q_t = at + bt^2 + ct^3,$$

show that the mean specific heat of the substance between t° and t'° is given by

$$S_t^{t'} = a + b(t + t') + c(t^2 + tt' + t'^2),$$

and that the true specific heat at $t°$ is

$$S_t = a + 2bt + 3ct^2.$$

41. Regnault found that 100·5 units of heat were required to raise the temperature of unit mass of water from 0° to 100°, and 203·2 to raise its temperature to 200°; taking the specific heat of water at 0° as unity (*i.e.* putting $a = 1$ in the preceding equations), prove that the true specific heat of water at any temperature $t°$, between 0° and 200°, is given by the equation

$$S_t = 1 + 0{\cdot}00004t + 0{\cdot}0000009t^2.$$

Show also that the true specific heat of water at 150° is 1·02625.

2. Change of State and Latent Heat.

Note.—The latent heat of water is 80, and the latent heat of steam is 536.

42. How would you show that heat is absorbed when common salt is dissolved in water?

Anhydrous calcium chloride eagerly absorbs water, and dissolves in it with evolution of heat; whereas when crystallised chloride of calcium is dissolved in water, the temperature falls. How do you explain these facts?

43. A quantity of common salt is mixed with water, both being at 0° C. After solution the temperature is found to be below 0°, and if melting ice be employed instead of water, the fall in temperature is still more marked. Explain these results.

44. A glass flask is filled with a mixture of ice and water, and a narrow tube with an india-rubber stopper is fitted into the neck so as to force the water up to a certain height in the tube; the flask is then immersed in lukewarm water. State exactly what will be observed in the tube.

45. Give a sketch of the reasoning by which it was predicted that the effect of a great increase of pressure would be to lower the melting-point of ice. Show how the prediction has been verified; and mention any important results arising from this fact.

46. If a fine wire with weights at its ends is hung over a block of ice at 0°, it is found that the wire cuts through the block, but that the ice reunites behind the wire, leaving the block still continuous. Explain this.

47. Define the latent heat of fusion of ice, and show that its numerical value may be deduced from the fact that when equal weights of boiling water and melting ice are intimately mixed, the ice all melts, and the resulting temperature is 10°.

48. How much ice at 0° will be melted by 500 grammes of boiling water?

49. How much hot water at 75° will just melt 10 lbs. of ice?

50. Would a change in the thermometric scale affect the numerical value of the latent heat of a substance? The specific heat of lead is 0·0315 and its latent heat of fusion is 5·34 when the Centigrade scale is employed: what are the corresponding numbers in the Fahrenheit scale?

51. How many grammes of ice must be dissolved in a litre of water at 20° in order to reduce its temperature to 5°?

52. 300 grammes of melting ice are mixed with 700 grammes of boiling water, and the resulting temperature is 46°: what is the latent heat of fusion?

53. How many pounds of iron (sp. heat 0·112) at 400° must be introduced into an ice calorimeter in order to produce 3 lbs. of water?

54. A 10-gramme weight made of brass (sp. heat 0·09) is heated to 100°, and dropped into a mixture of ice and water: how much ice will be melted?

55. In an experiment made to determine the latent heat of fusion of ice, 120 grammes of ice were dropped into a beaker containing 300 grammes of water at 50°. After all the ice was melted, the temperature of the water was found to be 13°: calculate the latent heat of fusion.

56. A ball of copper weighing 30 gm. was heated to 100° and placed in an ice calorimeter. In cooling down it evolved sufficient heat to melt 3·54 gm. of ice: what is the specific heat of copper?

57. 155 c.c. of water was obtained when a piece of iron at 100° was introduced into an ice calorimeter: if the mass of the iron was 1·08 kilogramme, what was its specific heat?

58. The bottom of a cylindrical vessel is covered by a layer of ice 1 decimetre thick. What must be the height of a column of boiling water which, when poured upon the ice, will just suffice to melt it? [Sp. gr. of ice $=0{\cdot}917$; sp. gr. of boiling water $=0{\cdot}96$.]

Let σ denote the sectional area (in sq. cm.) of the vessel, and h the height of the column of hot water. The volume of the water is $h\sigma$ c.c., its mass is $h\sigma \times 0{\cdot}96$, and the number of heat-units which it evolves in cooling from 100° to 0° is $h\sigma \times 0\ 96 \times 100$.

Again, the mass of the layer of ice is $10\sigma \times 0{\cdot}917$, and the number of heat-units required to melt it is $10\sigma \times 0{\cdot}917 \times 80$. Equating, we have

$$96h\sigma = 800 \times 0{\cdot}917\sigma,$$

and

$$h = 733{\cdot}6/96 = 7{\cdot}64.$$

Thus a column of water 7·64 cm. high will give out just enough heat to melt the ice.

59. Assuming that the specific heat of ice is 0·5, its specific gravity 0·92, and the weight of a cubic foot of water 62·5 lbs., find how many pound-degree units of heat are required to convert a block of ice 1 ft. long, 6 in. thick, and 9 in. broad, at −10° into steam at 100°.

60. 1 cwt. of ice at 0° was taken into a warm room: some time afterwards it was found to have melted completely, and the water produced by the fusion was at 21°. Express the cooling effect of the ice in pound-degrees of heat.

If the air in the room mentioned was originally at 29° and finally at the temperature of the water, find how many pounds of air were cooled from 29° to 21° by the changes mentioned, taking the specific heat of air as 0·237.

61. Find the result of mixing 1 lb. of snow at 0° with 4 lbs. of water at 30°.

When snow (or ice) is mixed with water, one of two things must happen: either a portion only of the snow will be melted, in which case the mixture of snow and ice will have a temperature of 0°; or the whole of the snow will be melted, in which case the final temperature can be found as below. In solving such questions the student should first find by inspection whether the whole or part only is melted; otherwise he will only be able to form from the given data a single equation to find two unknown quantities.

The amount of heat required to melt 1 lb. of snow at 0° is 80 pound-degree units of heat, and the number of these heat-units which 4 lbs. of water would evolve, in cooling from 30° to 0°, is $4 \times 30 = 120$; since $120 > 80$, it is clear that the whole of the snow will be melted. Let θ be the final temperature of the mixture: the heat evolved by the water is $4 \times (30 - \theta)$, and the heat absorbed by the snow (first in melting, and then in being raised to $\theta°$) is $80 + \theta$. Equating, we have

$$120 - 4\theta = 80 + \theta,$$

and

$$\theta = 40/5 = 8.$$

Thus the result is 5 lbs. of water at 8°.

62. Find the result of mixing 2 lbs. of ice at 0° with 3 lbs. of water at 45°.

Only a portion of the ice will be melted, for the amount of heat which the water can give out in cooling to 0° is only $3 \times 45 = 135$; whereas $2 \times 80 = 160$ heat-units are required to melt all the ice. Thus the final temperature will be that of a mixture of ice and water, viz. 0°. If x denote the amount of ice melted,

$$80x = 3 \times 45, \text{ and } \therefore x = 1 \cdot 7.$$

$$\text{Result} \left\{ \begin{array}{lll} 4 \cdot 7 & \text{lbs. of} & \text{water} \\ 0 \cdot 3 & ,, & \text{ice} \end{array} \right\} \text{all at } 0°.$$

63. If 5 lbs. of snow be mixed with 2 lbs. of water at 60°, how much snow will be melted?

64. Explain clearly what is meant by the statements: "the specific heat of ice is 0·5," "the latent heat of water is 80." What is the result of mixing a pound of ice at $-10°$ with a pound of water at 50°?

65. If a quantity of snow be dissolved in five times as much water at 25°, by how much will the temperature of the water be lowered?

66. Find the result of mixing 3 lbs. of melting ice with 7 lbs. of water at 60°.

67. What will happen if 2 lbs. of boiling water are poured upon 2 lbs. of snow at 0°?

68. Phosphorus melts at 44°·2, but can be cooled considerably below this temperature before solidification sets in. Show that if solidification commences at a temperature of 30°·7, just one-half of the mass will remain in the molten state. The latent heat of fusion of phosphorus is 5·4, and its specific heat in the molten state is 0·2.

Let the quantity of melted phosphorus be denoted by unity, and let x be the amount which solidifies: on solidification the temperature of the whole rises suddenly, the result being a mass x of solid phosphorus, and a mass $(1 - x)$ of molten phosphorus, both at 44°·2. The amount of heat evolved in the solidification is $5 \cdot 4x$, and the amount required to raise the whole mass to 44°·2 is $(44 \cdot 2 - 30 \cdot 7) \times 0 \cdot 2$. Equating these quantities, we have $5 \cdot 4x = 13 \cdot 7 \times 0 \cdot 2$, and $x = 2 \cdot 7/5 \cdot 4 = 0 \cdot 5$, so that exactly one-half of the phosphorus remains in the molten state.

69. 10 gm. of phosphorus is melted and then gradually cooled to 26°, at which temperature solidification commences: find how much will remain in the liquid state.

70. A quantity of water is cooled down gradually to -15°, and it then commences to freeze: assuming that the specific heat of water below the freezing point is unity, find how much ice will be produced.

71. Discuss fully the relative advantages and disadvantages of the various methods of determining the specific heat of a substance, stating which you would adopt when the amount of the substance at your disposal is very small. How would you proceed to determine the specific heat of a substance by means of Bunsen's ice-calorimeter?

Theory of the Bunsen Ice-calorimeter.—The latent heat of fusion of ice is 80, *i.e.* 80 heat-units are required to melt 1 gramme of ice. Now since the density of ice is 0·91674, 1 c.c. of ice weighs 0·91674 gramme; hence the volume of a gramme of ice is $1/0{\cdot}91674 = 1{\cdot}0908$ c.c., and a mixture of ice and water will diminish in volume by 0·0908 c.c. for each gramme of ice melted. The application of a single unit of heat to such a mixture will cause a diminution in volume of $0{\cdot}0908/80 = 1/881$ c.c.; in other words, 881 heat-units are required to cause a contraction of 1 c.c.

Let a body of mass m, and specific heat s, be heated to t° and dropped into the calorimeter; in cooling down to 0° it will give out mst units of heat. If the observed diminution in volume of the mixture of ice and water be denoted by v, then $881v$ heat-units must have been evolved. Equating these two quantities, we have,

$$ms\theta = 881v,$$

an equation to find s.

[The value given above for the density of ice,—0·91674,—is that obtained by Bunsen. Bunsen also made a special determination of the latent heat of fusion of ice, and found it to be 80·025; other important determinations have given

79·24 (Regnault), and 79·25 (Person, De la Provostaye and Desains). The calculation is not quite correct, because we have assumed that a gramme of water at 0° has unit volume, whereas its volume is really somewhat greater, viz. 1·00012 c.c. In solving the examples which follow, the value of the constant (881) should not be assumed, but each problem should be worked out from the given data.]

72. 0·484 gm. of a metal at 100° is dropped into a Bunsen's calorimeter, and the thread of mercury moves backwards through 1·21 cm. in the capillary tube, the diameter of which is 0·6 mm.: assuming that 881 heat-units are required to cause a contraction of 1 c.c., show that the specific heat of the metal is 0·06227.

Note.—In solving Examples 73-77 the volume of a gramme of ice is to be taken as 1·09 c.c., and the latent heat of water as 80.

73. What change will be produced in the volume of a mixture of ice and water when 150 heat-units are imparted to it?

74. A mixture of ice and water was placed in a test-tube and occupied a volume of 30 c.c.; the test-tube was held in hot water until the volume had diminished to 29 c.c.: how much heat had been absorbed in the meantime?

75. A 10-gramme weight made of brass (sp. heat = 0·09) is heated to 100° and dropped into an ice-calorimeter: what contraction will it produce?

76. What change of volume would be produced in an ice-calorimeter by placing in it 2·5 gm. of a substance of specific heat 0·076 at a temperature of 100°?

77. Calculate the specific heat of a metal which gave the following results, according to Bunsen's method:—0·96 gm. was heated to 99°·5, and dropped into the calorimeter; the thread of mercury retreated through a distance of 8·3 mm. in the capillary tube, whose bore had a cross-section of 1 sq. mm.

78. 10 grammes of water at 88° are placed in the inner tube of a Bunsen's calorimeter, and it is found that the volume of the contents of the outer portion decreases by exactly 1 c.c.: taking the latent heat of water as 80, what value does this give for the specific gravity of ice?

79. 0·87 gramme of a substance was heated to 98°·6, and then dropped into a Bunsen's ice-calorimeter; the movement of the mercury column showed that the volume of the mixture of ice and water surrounding the body had diminished by 7·9 cub. mm: find the quantity of ice melted, and the specific heat of the substance from the data given. [Latent heat of fusion of ice = 80. Sp. gr. of ice = 0·917.]

80. It is said that the vapour of any liquid exerts a definite pressure which depends upon (1) the nature of the liquid, and (2) the temperature: how would you proceed to show that this is the case?

The surface of a liquid is exposed to the atmosphere, and heat is gradually applied until the pressure of the vapour of the liquid becomes equal to the atmospheric pressure: explain fully what occurs.

81. What do you understand by the expression "maximum tension of water vapour" at a given temperature? How do you explain the existence of a maximum vapour-pressure at every temperature, and why does it increase with the temperature?

82. Describe briefly the methods which you would employ for measuring the maximum pressure of water-vapour (1) below 60°, (2) between 60° and 100°, and (3) above 100°.

83. How would you find by experiment (1) the minimum pressure at which a very volatile liquid such as liquified sulphur dioxide can be kept exposed without boiling, and (2) the maximum pressure at which common ether will begin to boil, both substances being supposed to be kept at the atmospheric temperature?

84. Define the boiling point of a liquid, and distinguish between ebullition and evaporation. What condition determines whether a liquid will boil or evaporate?

85. State the laws which govern the phenomenon of boiling, and mention any abnormal cases in which these laws are not followed.

What precautions would you take (1) in order to make a liquid boil very regularly, (2) to make it boil at as low a temperature as possible?

86. Water at 15° is sprinkled upon the floor of a chamber which contains dry air at 15° and 76 cm. pressure: how will the pressure be affected (1) if the chamber is air-tight, (2) if there is free communication with the atmosphere? Will the temperature be affected at all? (The vapour pressure of water at 15° is 1·27 cm.)

87. The table below gives the maximum pressure of water-vapour in millimetres of mercury:—

4° . .	6·1	8° . .	8·0	12° . .	10·4
6° . .	7·0	10° . .	9·1	14° . .	11·9

State briefly how these numbers have been obtained; and find the actual pressure of the water-vapour present in a room at 14° when the dew-point is found to be 5°. What is the relative humidity of the air in the room?

88. Calculate the latent heat of steam from the results of the following experiment, allowing for the heat absorbed by the brass calorimeter:—

Weight of calorimeter	326·3	gm.
,, ,, + water . .	757·7	,,
,, steam condensed . . .	46·35	,,
Temperature of steam . . .	100°	
,, water before experiment	7°·5	
,, ,, after experiment .	65°·2	

Taking the specific heat of brass as 0·09, the water-equivalent of the calorimeter is 326·3 × 0·09 = 29·4 (q. p.)

The steam in condensing gives out $46{\cdot}35x$ units of heat, x denoting the required value of the latent heat of steam. Further, in cooling from 100° to 65°·2 it evolves $46{\cdot}35 \times 34{\cdot}8 = 1612{\cdot}6$ heat-units. These are together equal to the amount of heat required to raise the water and calorimeter from 7°·5 to 65°·2, *i.e.* to

$$(431{\cdot}4 + 29{\cdot}4)(65{\cdot}2 - 7{\cdot}5) = 460{\cdot}6 \times 57{\cdot}7 = 26588{\cdot}2.$$

Thus $$46{\cdot}35x + 1612{\cdot}6 = 26588{\cdot}2,$$

and $$x = 24975{\cdot}6/46{\cdot}35 = 539{\cdot}1.$$

89. The condenser of a steam-engine is supplied with injection water at a temperature $t°$, and the steam enters the condenser at 100°. How many pounds of injection water must be supplied for every pound of steam condensed, in order that the water may leave the condenser at a temperature T° ?

1 lb. of steam in condensing to water at 100° sets free 536 units (pound-degrees) of heat, and in cooling from 100° to T° it further gives out (100 − T) heat-units. These are together equal to $x(T - t)$, the amount of heat absorbed by the injection-water, x being the required weight of water, which is heated from $t°$ to T°.

Thus $$x(T - t) = 536 + (100 - T),$$

and $$x = (636 - T)/(T - t).$$

90. What is meant by saying that the latent heat of steam is 536? How many heat-units are required to convert 50 grammes of water at 12° into steam at 100° ?

91. A vessel containing 30 gm. of ice is placed over a spirit-lamp: how much heat will be required to melt it and vaporise the water completely ?

92. How many pounds of steam at 100° will just melt 50 pounds of ice at 0° ?

93. 10 gm. of steam at 100° is condensed in a kilogramme of water at 0°, and the temperature of the water is thereby raised to 6°·3 : what value does this give for the latent heat of steam ?

94. How many grammes of steam at 100° must be passed into 200 gm. of ice-cold water in order to raise it to the boiling-point? What will happen if more steam than this is passed in?

95. Compare the amount of heat required to convert a given mass of ice at – 3°·2 into water at 38°, with that required to convert the same mass of water at 38° into steam at 100°. [The specific heat of ice is 0·5.]

96. A kilogramme of cold ice is taken at – 10°, and heat is continually applied to it until a temperature of 1000° is attained: trace the successive effects produced, stating the amount of heat required for each of these effects.

97. Calculate the latent heat of steam from the results of the following experiment, made with the same apparatus as that of Ex. 88:—

Weight of calorimeter	326·3	gm.
,, ,, + water	915·3	,,
,, steam condensed	28·38	,,
Temperature of steam	100°	
,, water before experiment	2°·8	
,, ,, after experiment	30°·8	

98. A calorimeter weighing 150 gm., and made of silver of specific heat 0·056, contains 350 gm. of water at 8°. If 10 gm. of steam at 100° is passed into the water, what will be the temperature after equilibrium has been attained, supposing no heat to be lost or gained?

EXAMINATION QUESTIONS.

99. Define the notion of *temperature.*

The temperature of a fluid is ascertained by means of the hand to be the same as that of a mixture of 3 lbs. of water at 0° C. with 7 lbs. of water taken at 100° C. What is the temperature of the fluid?

Edinb. M.A. 1882.

100. Compare the advantages of mercury and alcohol as thermometric substances.

It is sometimes stated that the low specific heat of mercury (·033) is one of its special qualifications. Show that if the coefficients of expansion of the two liquids were the same, and two similar thermometer tubes were filled, one with each liquid, that filled with alcohol would require a greater quantity of heat to raise it through the same range of temperature unless the specific heat of alcohol were less than ·528. [Sp. gr. of mercury 13·6; of alcohol ·85.] Camb. Schol. 1883.

101. A copper vessel containing a thermometer is at 12° C. 50 gm. of water at 60° are poured in, and the temperature after stirring is found to be 50°: find the thermal capacity or water-equivalent of the vessel and thermometer. Vict. Int. 1885.

102. State clearly the distinction between temperature and heat.

Twenty pound-degrees of heat are communicated to a metal vessel weighing 8 lbs., and containing 10 lbs. of water. If the specific heat of the metal be $\frac{1}{10}$, in what proportion will the heat be divided between the water and the vessel, and what will be their rise of temperature? Matric. 1884.

103. A bath being required at a temperature of 37° C., twelve pailsful of cold water at 10° are thrown into it. Hot water from a cistern at 55° is now poured in until the temperature of the bath is 25° C. If the water remaining in the cistern be now heated until it boils, show that the bath will be at the required temperature when four pailsful of the boiling water have been poured in, although its temperature falls 2° C. whilst the water in the cistern is being heated to the boiling point.

Camb. B.A. 1883.

104. 200 grammes of water at 99° C. are mixed with 200 cub. cm. of milk of density 1·03 at 15° C., contained in a copper vessel of thermal capacity equal to that of 8 grammes of water, and the temperature of the mixture is 57° C. If all the heat lost by the water is gained by the milk and the copper, what is the specific heat of the milk? Matric. 1885.

105. What is the latent heat of fusion of a substance?

A pound of ice at 0° C. is thrown into 6 lbs. of water at 15° contained in a copper vessel weighing 3 lbs., and when the ice is melted the temperature of the water is 2° C: find the latent heat of fusion of ice, the specific heat of copper being 0·095. Matric. 1886.

106. Describe experiments illustrating the difference between temperature and heat.

In 100 grammes of boiling water ($t = 100$) there are placed 20 grammes of ice, and the temperature falls to 70° when the ice is just melted: what is the latent heat of fusion of ice, assuming no heat to be lost?

Matric. 1887.

107. Five hundred cubic centimetres of mercury at 56° C. are put into a hollow in a block of ice, and it is found that 159 grammes of the ice are liquified: find the specific heat of mercury. Glasgow M.A. 1882.

108. Three separate mixtures are made, namely—

(1) Water and snow,
(2) Water and salt,
(3) Snow and salt.

If all the materials were, before being mixed, at 0° C., which mixture will be at the highest temperature and which at the lowest? and why? Matric. 1885.

109. Calculate the cooling effect of a cube of ice 2 ft. in the side, taken at 0° C. and reaching 27° C. when its cooling power has been exhausted. (The coefficient of the expansion of water on changing into ice is $\frac{1}{11}$, and the number of pounds in 1 cubic foot of water is 62·4.) Edinb. M.A. 1882.

110. How many units of heat would cause a mixture of ice and water to contract by 50 cubic millimetres, if 100 cub. mm. of water at 0° C. become 109 cub. mm. on freezing? Matric. 1883.

111. Describe Bunsen's calorimeter. If 100 c.c. of water in freezing become 109 c.c. of ice, and the introduction of 20 grammes of mercury at 100° C., into a Bunsen's calorimeter cause the end of the column of mercury to move through 74 mm. in a tube one square millimetre in section, find the specific heat of mercury. (The heat required to melt one gramme of ice is 80 units.) Matric. 1884.

112. A gramme of ice at 0° contracts 0·091 c.c. in becoming water at 0°. A piece of metal weighing 10 grammes is heated to 50°, and then dropped into the calorimeter. The total contraction is ·063 c.c.: find the specific heat of the metal, taking the latent heat of ice as 80. Vict. B.Sc. 1886.

113. Explain the formation of dew and hoarfrost.

How do you account for the fact that a cloud is sometimes formed by the mixture of two quantities of air at different temperatures, although neither quantity is quite saturated before the admixture? Matric. 1884.

114. Explain the principles on which the hygrometric state of the atmosphere is deduced from observations of the wet and dry bulb hygrometer, and show how the "constant" of this instrument is experimentally found.

Int. Sc. Honours 1885.

115. Describe and discuss carefully the experiments you would make to determine the dew-point and the tension of aqueous vapour in the air.

Air from a space saturated with moisture is drawn into an aspirator of known volume through drying tubes: show carefully how, by means of this experiment and tables of the saturating tension of aqueous vapour, to determine the temperature of the space.

Camb. Schol. 1883.

116. What is meant by the statement that the latent heat of steam is 537?

One pound of saturated steam at 160° C. is blown into 19 lbs. of water at 0° C., and the resulting temperature is 32·765° C.: find the latent heat of steam at 160° C. Matric. 1883.

117. The specific heat of mercury is ·03. A pound of steam at 100° C. is made to pass into a vessel containing 300 lbs. of mercury initially at 0° C., the capacity for heat of the vessel being equal to 10 lbs. of water: what will be the temperature of the vessel and contents at the end of the experiment? Matric. 1884.

118. Distinguish between calorimetry and thermometry.

20 grammes of steam at 100° C. are condensed in a metal worm surrounded by 200 grammes of water at 10° C. If the water equivalent of the worm be 10 grammes, and the latent heat of steam be 536, determine the temperature to which the water is raised.

Matric. 1886.

119. 1 lb. of ice at – 10° C. is placed in a closed vessel, and steam at 100° C. passed into it. It is found, after making the necessary corrections, that when the ice is just melted the resulting water weighs 1·134 lbs.; and that when the temperature has risen to 100° C., it weighs 1·345 lbs. The specific heat of ice is ·5: find the latent heats of water and steam. Camb. B.A. 1878.

CHAPTER V

CONDUCTIVITY AND THERMODYNAMICS.

Conductivity.—The thermal conductivity (or coefficient of conductivity) of a substance is measured by the number of units of heat which pass in unit time across unit area of a plate whose thickness is unity, when its opposite faces are kept at temperatures differing by one degree. In the statement of this definition it is supposed that the flow of heat has become *steady*, and that the lines of flow of heat are perpendicular to the surfaces of the plate.

The thermal conductivity of a substance, in the C.G.S. system, is measured by the quantity of heat which flows per second, under these circumstances, across one square centimetre of a plate one centimetre in thickness, the opposite faces of the plate being kept at temperatures differing by 1° C.

The quantity of heat (H) which flows in a given time across a plate of given dimensions is inversely proportional to the thickness (d) of the plate, and is directly proportional to the area of its surface (s), to the difference of temperature (θ) between its opposite faces, and to the time t. If the thermal conductivity of the substance be denoted by k,

$$H = k \cdot \frac{s\theta t}{d} \quad . \quad . \quad . \quad . \quad (1)$$

1. A large tank is covered with a layer of ice 6 cm. thick and 24 square metres in area: assuming that the

coefficient of conduction of ice in C.G.S. units is 0·0057, determine the amount of heat transmitted per hour from the water up through the ice, the upper surface of which is at the temperature of the air, viz. $-10°$ C.

In order that the answer may be given in gramme-degrees of heat, we must express all quantities in terms of the corresponding C.G.S. units. Thus $s = 24$ sq. metres $=$ 240,000 sq. cm., and $t = 60$ minutes $=$ 3600 seconds.

Substituting these values in equation (1), we have

$$\begin{aligned} H &= 0{\cdot}0057 \times 240{,}000 \times 10 \times 3600/6 \\ &= 8{,}208{,}000 \text{ units.} \end{aligned}$$

2. The top of a steam chamber is formed of a stone slab 6 decimetres long, 5 decimetres broad, and 1 decimetre thick. Ice is piled upon the slab, and it is found that 5 kilogrammes of ice are melted in half an hour: what is the thermal conductivity of the stone?

Since the latent heat of fusion of ice is 80, the amount of heat required to melt 5 kilogrammes of ice is

$$H = 5 \times 1000 \times 80 = 400{,}000 \text{ heat-units.}$$

This amount of heat is transmitted in 1800 seconds through a slab of $60 \times 50 = 3000$ sq. cm. area and 10 cm. thickness, its opposite faces being kept at 0° and 100° degrees respectively. Thus

$$400{,}000 = k \times 3000 \times 100 \times 1800/10,$$

and

$$k = 4/540 = 0{\cdot}00741.$$

3. The coefficient of conduction of copper is 0·96: how many heat-units will pass per minute across a plate of copper 1 metre long, 1 metre broad, and 1 cm. thick, when its opposite faces are kept at temperatures differing by 10°?

4. The wall of a cottage is 2 decimetres thick, and is built of stone whose thermal conductivity is 0·008; the temperature inside the cottage is 18°, while the outside temperature is 2°: how much heat is lost by transmission per hour across each square metre of the wall?

5. It is found that $1{\cdot}44 \times 10^7$ heat-units are transmitted per hour across an iron plate 2 cm. thick and 500 sq. cm. in area, when its opposite sides are kept at 0° and 100° respectively: what is its coefficient of conductivity?

6. An iron boiler is made of plate 0·8 cm. thick, and its total surface is 8 square metres: the water inside is at a temperature of 120°, and the external surface of the boiler is at 95°. Assuming that the thermal conductivity of the iron is 0·164, find how much heat is lost by conduction per hour.

7. A metal plate, 1 sq. decimetre in area and 0·5 cm. thick, has the whole of one face covered with melting ice, while the other face is in contact with boiling water. The coefficient of conductivity of the metal is 0·14: how many kilogrammes of ice will be melted in an hour?

8. "In a solid, heat may be transmitted from point to point in two ways, and in a fluid in three ways." Discuss this statement, and comment upon the following facts: When a sheet of glass is held in front of a hot stove it appears to cut off the heat given out by the stove; but when the sun shines upon the glass windows of a greenhouse the heat passes readily through without producing any considerable change in the temperature of the glass itself. Explain carefully how it is that the air inside the greenhouse may in this way become hotter than the outside air.

9. An iron vessel containing a kilogramme of ice is partially immersed in a tank of water at 15°, so that the total area of the immersed surface is 400 sq. cm. The mean thickness of the wall of the vessel is 0·8 cm., and exactly one minute after immersion all the ice is found to have melted. Calculate from this the thermal conductivity of iron, and discuss the validity of any assumptions made in your solution.

10. Péclet states that the quantity of heat which passes in an hour through a plate of lead 1 square metre in area and 1 metre thick, with a difference of 1° between the temperatures of its surfaces, is 13·83 kilogramme-degrees: what value does this give for the C.G.S. coefficient of conductivity for lead?

11. Point out the experimental difficulties which would be met if you attempted to carry out practically the idea contained in the definition of conductivity given on p. 137. Describe the principle and the general results of Forbes's experiments, indicating how the calculations were made; and find the value of the multiplier for reducing to the C.G.S. system coefficients of conductivity expressed in kilogramme-degrees per square metre, per millimetre, per second.

Mechanical Equivalent of Heat.—The amount of work which is equivalent to one heat-unit is called the mechanical equivalent of heat. Its value was determined by Joule (whence it is sometimes called "Joule's equivalent"), and is denoted by the letter J.

Joule found that 772 foot-pounds of work were required to raise the temperature of a pound of water through one degree Fahrenheit; the corresponding number in terms of the degree Centigrade is $772 \times 9/5$, or 1389·6. Since 1 foot = 30·48 cm., the mechanical equivalent is $1389{\cdot}6 \times 30{\cdot}48 = 42355$ gramme-centimetres per gramme-degree, or about 424 kilogramme-metres per kilogramme-degree of heat.[1] To convert this into absolute measure we have to multiply by g ($=981$), which gives 42355×981, or $4{\cdot}155 \times 10^7$ ergs per gramme-degree.

More recent experiments have shown that the number which we have given is somewhat too low. As the third significant figure of this important constant has not been determined with any accuracy, we shall adopt in our calculations the approximate value

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[1] Observe that the value of J is not affected by a change in the unit of *mass* employed, for it is a ratio between an amount of work and a corresponding amount of heat, and the unit of mass is involved in the same degree (*i.e.* to the first power) in the units of work and heat. On this account the usual statement that the "mechanical equivalent of heat is 1390 foot-*pounds*" is somewhat misleading.

$$J = 4{\cdot}2 \times 10^7.$$

[It should be noticed that all the above numbers—772, 424, etc. —with the exception of the last, give the value of J *in gravitation measure*, and care should be taken to make the required conversion into absolute measure (or *vice versa*) where it is necessary. In working problems relating to potential energy, or the energy of bodies falling from a height, it is convenient to use gravitation measure; but it is preferable to work in absolute measure throughout.]

The energy of a body of mass m moving with velocity v is $mv^2/2$ in dynamical measure (pp. 35-38). The thermal equivalent of this is $mv^2/2J$. Suppose the body to meet an obstacle and to fall dead. Also let s denote its specific heat, and let us assume that all the heat developed by its impact goes to raise its temperature through (say) $\theta°$. The amount of heat required to produce this rise of temperature is $ms\theta$. Thus

$$mv^2/2J = ms\theta,$$

and

$$\theta = v^2/2Js.$$

Similarly, if the body be raised to a height h above the ground, its potential energy is mgh ergs, and the thermal equivalent of this is mgh/J. If the body falls to the ground, and if we suppose that all the heat produced by the arrest of its motion is spent in warming it, the rise of temperature produced will be

$$\theta = gh/Js.$$

In both cases the elevation of temperature is independent of the mass of the body.

12. A leaden bullet of specific heat 0·032 strikes against an iron target with a velocity of 400 metres per second. If the bullet falls dead, and the heat produced is equally divided between it and the target, find its temperature, supposing it originally at 10°.

If the mass of the bullet be m, its kinetic energy is $(40{,}000)^2 \times m/2$, and the equivalent of this in heat-units is

$$(40{,}000)^2 \times m/2 \times 4\cdot2 \times 10^7 = 80m/4\cdot2.$$

Half of this goes to heat the bullet. Suppose the rise of temperature produced is $\theta°$, then

$$40m/4\cdot2 = m \times 0\cdot032\,\theta,$$

and $$\theta = 40/4\cdot2 \times 0\cdot032 = 298.$$

Since the bullet was originally at 10°, its temperature after striking the target is 308°.

13. In obtaining work from an engine at the rate of 20 H.P., 56 lbs. of coal are consumed per hour: find the efficiency of the engine, assuming that the heat produced by the combustion of 1 lb. of coal is sufficient to convert 15 lbs. of water at 100° into steam at the same temperature.

The *efficiency* of a steam engine is the ratio between the useful work performed and the work represented by the heat of combustion of the fuel.

The work done per minute by the engine is

$$20 \times 33{,}000 = 660{,}000 \text{ ft.-lbs.}$$

The combustion of 1 lb. of coal produces

$$15 \times 536 = 8040 \text{ thermal units.}$$

Since 56 lbs. of coal are consumed per hour, the amount of heat evolved per minute is

$$56 \times 8040/60 = 7504,$$

which is equivalent to $7504 \times 1390 = 10{,}430{,}560$ ft.-lbs. Thus the efficiency of the engine is

$$E = 66000/10430560 = 0\cdot06328$$
(or 6·328 per cent).

14. How much heat is set free when a body of mass

100 grammes, moving at the rate of 25 metres per second, is suddenly brought to rest?

15. From what height must a raindrop fall to the ground in order that its temperature may be raised 1° C.? [Take J = 1390.]

16. Find the equivalent in ergs of the amount of heat required to raise 1 lb. of water through 1° C.

17. A block of ice is dropped into a well of water, both ice and water being at 0°. From what height must the ice fall in order that one-fiftieth of it may be melted? [Take J = 424.]

18. Mercury falls from a height of 10 metres upon a perfectly non-conducting surface. How much warmer will it be after the fall? [Sp. heat = 0·033.]

19. How many heat-units are required to raise the temperature of a kilogramme of iron [sp. heat = 0·112] through 40°? If the equivalent amount of kinetic energy were imparted to it, what would be its velocity?

20. The heat of combustion of hydrogen is 34,460 calories, *i.e.* 1 gramme of hydrogen in burning gives out enough heat to raise the temperature of 34,460 grammes of water through 1°. Express in Watts (§ 3) the power which would be obtained if the heat produced by the combustion of 50 grammes of hydrogen per hour were completely converted into work.

21. Power equal to 100 Watts is entirely converted by friction into heat: how much water can be heated in this manner through 1° in an hour?

22. In one of Rumford's experiments on the boring of brass cannon, the heat developed by a horse working for $2\frac{1}{2}$ hours was found to be sufficient to raise the temperature of 26·5 lbs. of water from 0° to 100°. Calculate the number of foot-pounds of work done, and the rate at which the horse worked.

23. A cannon-ball moving at the rate of 800 feet per second strikes against a target, and the heat produced is equally divided between the target and the ball:

supposing the latter to be made of iron of specific heat 0·112, prove that its temperature will be raised by 32°.

24. Lead melts at 335°, and its latent heat of fusion is 5·4. Taking its mean specific heat as 0·032, calculate the equivalent (in ergs) of the amount of heat required to raise the temperature of 10 grammes of lead from 0° to its melting point, and to melt it.

25. With what velocity must a leaden bullet, at a temperature of 15°, strike a target in order that the heat produced may be just sufficient to melt it?

26. Calculate the efficiency of a steam engine from the following details:—

Diameter of cylinder	16 in.
Length of stroke	2 ft. 6 in.
Number of revolutions per minute .	100
Mean pressure on piston . . .	22 lbs. per sq. in.
Consumption of coal . . .	1 cwt. per hour.

[Assume the data given in Example 13, and see Chap. I. Example 128.]

Work done in Expansion.—Let a cylinder containing air be closed by a weightless movable piston, and let a be the area of the piston, and p the pressure upon it per unit of area: the whole pressure upon the piston is $P = pa$. Now let the air expand and force the piston through a distance d against this constant pressure. The work done is the product of the force into the distance through which it is overcome, or

$$W = P \cdot d = pa \cdot d.$$

But $a\,d$ is the volume of the space through which the piston has moved, *i.e.* the change of volume produced by expansion of the air. Denoting this by dv, we have

$$W = p \cdot dv,$$

or the work done in expansion is equal to the pressure per unit area multiplied by the increment of volume.

If the work done is to be expressed in ergs, the pressure (p) must be measured in dynes per square centimetre, and the change of volume (dv) in cubic centimetres. When the expansion is caused by heat, and the original volume and initial and final temperatures are known, the change of volume can be calculated by applying Charles's law. If the barometric height (h) be given, the atmospheric pressure in terms of the above units (see p. 70) is

$$\Pi = h\rho g,$$

or, taking the normal barometric height of 76 cm.,

$$\Pi = 76 \times 13{\cdot}596 \times 981 = 1{,}013{,}226.$$

To find the specific heat of air at constant volume when its specific heat at constant pressure and the mechanical equivalent of heat are known.

Let m denote the density of air (or its mass per unit volume), C its specific heat at constant pressure, and c its specific heat at constant volume.

Suppose 1 c.c. of air taken at 0° to be heated to 273° under the constant atmospheric pressure Π. The amount of heat absorbed is

$$H = 273\, mC.$$

Part of this is spent in doing external work (p. 144), for during the process the volume of the air is doubled ; the change of volume (dv) is 1 c.c., and the work done during the expansion is Π ergs. The thermal equivalent of this is

$$h = \Pi / J.$$

Now if the temperature of the air had been raised to 273° without allowing it to do any external work (*i.e.* if it had been heated at constant volume), the amount of heat absorbed would have been

$$H' = 273\, mc,$$

and it is clear that

$$H = H' + h,$$

or

$$H' = H - h.$$

Thus $$273mc = 273mC - \Pi/J,$$
and the specific heat of air at constant volume is

$$c = C - \Pi/273mJ.$$

Now we have seen that $\Pi = 1{,}013{,}226$ (p. 145), and $J = 4{\cdot}2 \times 10^7$ (pp. 140, 141). Also $C = 0{\cdot}237$, and $m = 0{\cdot}001293$, so that the numerical value of c is

$$0{\cdot}237 - 1013226/273 \times 0{\cdot}001293 \times 4{\cdot}2 \times 10^7 = 0{\cdot}237 - 0{\cdot}068$$
$$= 0{\cdot}169.$$

The ratio between the two specific heats is

$$\gamma = 0{\cdot}237/0{\cdot}169 = 1{\cdot}4.$$

27. In one of Joule's earlier experiments air was compressed into a copper receiver standing in a calorimeter, the water equivalent of the calorimeter and its contents being 10,682 gm.; the air was then allowed to escape slowly, and was found to measure 44·6 litres, the atmospheric pressure being 76·5 cm. The cooling effect in the calorimeter was 0°·097: what value does this give for the mechanical equivalent?

The work done (in ergs) is

$$W = \Pi \cdot dv = h\rho g \cdot dv$$
$$= 76{\cdot}5 \times 13{\cdot}596 \times 981 \times 44600 = 4{\cdot}55 \times 10^{10}.$$

The corresponding amount of heat absorbed is

$$H = 10682 \times 0{\cdot}097 = 1{\cdot}036 \times 10^3 \text{ heat-units.}$$

This gives for the mechanical equivalent of heat

$$J = W/H = 4{\cdot}55 \times 10^7/1{\cdot}036 = 4{\cdot}39 \times 10^7.$$

28. A receiver provided with a stop-cock contains air at a pressure p, which is greater than the atmospheric pressure Π; the temperature of the air is the same as that of the atmosphere, viz. t. The stop-cock is opened, and is shut again as soon as the air inside is at the atmospheric pressure: when the air has again attained the temperature t will its pressure be greater or less than Π? Assuming this pressure to be p', determine the temperature at the moment when the stop-cock is closed, and find how much air escapes.

The air escapes from the receiver until its pressure is reduced to that of the atmosphere (Π); in expanding it does work against the external pressure of the atmosphere, and consequently it is cooled to some temperature t' lower than t. At this temperature its pressure is Π, but when it is again at t, its pressure will have increased to p'; and if a denote the coefficient of increase of pressure of air,

$$\Pi : p' = 1 + at' : 1 + at,$$

and $$1 + at' = \Pi(1 + at)/p',$$

or, if T' and T are the absolute temperatures corresponding to t' and t,

$$T' = \Pi T/p'.$$

When the pressure in the receiver is p' it contains p'/p of the amount of air contained when the pressure was p; hence $(p - p')/p$ of the original amount of air escapes.

29. A cubic metre of air is heated under the normal pressure until its volume is doubled: how much of the heat supplied to it is converted into external work?

30. In the transmission of energy by means of compressed air, the air is compressed by means of a condensing pump until its pressure is 8 atmospheres. Find the percentage loss of energy which should theoretically occur owing to the cooling of the air to its initial temperature. Would you expect the actual loss in the whole process of transmission to be greater or less, and why?

31. Explain what is meant by an indicator diagram, and draw (to any scale) the isothermals for a perfect gas at 0°, 10°, and 20°. Show in your figure the lines that would be traced if the gas were heated (*a*) at constant pressure from 10° to 20°, (*b*) at constant volume through the same range of temperature. How would you represent on the diagram the mechanical equivalents of the quantities of heat required for the two operations?

32. Define the critical temperature of a fluid. Give rough sketches showing the probable forms of the isothermals for a body both above and below its critical point; and specify the most important distinction between the two classes of isothermals.

33. A gramme of air is heated from 0° to 100° under a pressure of 76 cm.: how much external work is done in the expansion?

34. A cubic foot of air is heated from 0° to 273°, the barometric height at the time being 30 inches (p. 70): how many foot-pounds of work are done during the expansion? and what is the heat equivalent of this work?

35. Explain precisely what is meant by the *efficiency* of an engine. What do you understand by a *reversible* engine, and what are the conditions of reversibility in a heat engine?

Would you expect to get more work from 1 lb. of water in cooling from 100° to 0°, or from 100 lbs. of water in cooling from 1° to 0°, and why?

36. The density of air at 0° and 76 cm. is 0·001293; its specific heat at constant pressure is 0·237, and its specific heat at constant volume is 0·169. Calculate from these data the value of the mechanical equivalent of heat.

37. Write a short essay on temperature, starting from the elementary ideas of "hot" and "cold," and criticising the methods commonly adopted for the measurement of temperature. Explain what is generally meant by *absolute temperature*, and indicate the process by which it has been rendered possible (in a stricter sense) to measure temperatures absolutely.

EXAMINATION QUESTIONS.

38. Define the absolute conductivity of a substance. A metal plate $\frac{1}{4}$ inch in thickness and 2 feet square has the whole of one face in contact with water which is kept boiling, while the other face is in contact with melting ice; and it is found that 300 lbs. of ice are melted in one hour. Find the absolute conductivity of the metal, stating clearly the units you employ. Matric. 1879.

39. What is meant by the statement that the thermal conductivity of iron is ·2 C.G.S. unit? Given an iron bar 3 metres long and 2·5 cm. square in section, describe fully the experiments necessary to determine its thermal conductivity. How would you show that cold iron conducts better than hot iron? Int. Sc. Honours 1887.

40. The amount of work which can be derived from the consumption of a kilogramme of coke is 309×10^{12} ergs. Of the work thus derived one-twentieth is usefully employed in drawing trucks up a slope of 30°: find the amount of coke required to draw a weight of 9400 megadynes a distance of 1500 metres along the slope [1 megadyne $= 10^6$ dynes]. Ind. C. S. 1885.

The force acting along the slope is $9400 \times \cos 60° = 4700$ megadynes. The work done in overcoming this resistance through 1500 metres is

$$4700 \times 10^6 \times 150{,}000 = 705 \times 10^{12} \text{ ergs.}$$

Now the useful work done per kilogramme of coke consumed is

$$\frac{1}{20} \text{ of } 309 \times 10^{12} = 15{\cdot}45 \times 10^{12} \text{ ergs,}$$

hence the amount of coke required is

$$705/15{\cdot}45 = 45{\cdot}63 \text{ kilogrammes.}$$

41. What is the mechanical equivalent of heat? An engine consumes 40 lbs. of coal of such calorific power that the heat developed by the combustion of 1 lb. is capable of converting 16 lbs. of water at 100° C. into steam at the same temperature, and during the process the engine performs 16,000,000 foot-pounds of work: what percentage of the heat produced is wasted?

Int. Sc. 1881.

42. What is a heat engine, and what is a reversible heat engine? What condition must be fulfilled respecting the passage of heat to and from the working substance in order that a heat engine may be reversible?

What is the efficiency of an engine which consumes 28 lbs. of coal in drawing a train one mile against a resistance equal to the weight of 1½ ton, the calorific power of the coal being such that one pound is capable of converting 16 lbs. of boiling water into steam at the same temperature? Int. Sc. 1882.

43. When temperatures are expressed on the Centigrade scale, the latent heat of fusion of ice is represented by 80, and the mechanical equivalent of heat by 423·9 (metre grammes). Express the same quantities on the Fahrenheit scale, and explain why one is represented by a larger and the other by a smaller number.

Int. Sc. 1885.

44. A windmill works at 4 horse-power for 24 hours, and 90 per cent of the work done is stored up as energy. Of this energy 90 per cent is employed in heating water from 12° C. to the boiling point. How many pounds of water will be so heated? B. Sc. 1881.

45. State the gaseous laws, describing carefully the experiments you would adopt to verify them.

The specific heat of air at a pressure of 10^6 dynes being ·24, its specific gravity ·0012, and its coefficient of expansion ·0036, find the external work done by the application of one unit of heat to a c.c. of air in a cylinder with a movable weightless piston.

Camb. Schol. 1883.

46. Gases are generally said to possess two specific heats. Is there any reason for this distinction, and, if so, what is it? A quantity of air is heated, and by diminishing the pressure upon it, it is allowed to expand by one per cent of its volume at 0° C. for every increase of temperature of 1° C. Show how its specific heat under these conditions may be determined, and state whether it is greater or less than its specific heat at constant pressure. Int. Sc. Honours 1883.

47. How can the amount of work done against external pressure during change of volume be expressed numerically?

One gramme of air is heated under constant pressure from 0° to 10° C. Determine the work, either in ergs or in centimetre-grammes, due to the expansion.

[Coefficient of expansion of air, $\frac{1}{273}$. Volume of 1 gramme of air at 0°, under pressure of 1 million dynes per square centimetre = 783·8 cubic centimetres. Or, 1 cubic centimetre air at 0° under pressure of 76 cm. mercury = 0·001293 gramme; 1 cubic centimetre mercury at 0° = 13.596 grammes.; g = 981 (centim., seconds)].
Int. Sc. 1884.

48. Calculate the mechanical equivalent of heat from the following data for oxygen:—

Coefficient of expansion, 1/273.
Mass of a litre at 0° and 760 mm., 1·43 gm.
Specific heat for constant pressure referred to an equal mass of water, ·22.
Ratio of the specific heat at constant pressure to the specific heat at constant volume, 1·4/1.
Specific gravity of mercury, 13·6.

Balliol Coll. 1882.

CHAPTER VI

LIGHT

1. If the light of the full moon is found to produce the same degree of illumination as a standard candle does at a distance of 4 ft., what is the equivalent in candle-power of the moon's light? (Take the distance of the moon as 240,000 miles.)

Let E denote the equivalent in candle-power. The degree of illumination is inversely proportional to the square of the distance, and since the illuminating power of the candle is taken as unity,

$$E/(240{,}000 \times 1760 \times 3)^2 = 1/4^2,$$
$$\therefore\ E = (240{,}000 \times 1760 \times 3/4)^2,$$
$$= 10{,}036{,}224 \times 10^{10}.$$

2. A standard candle and a gas-flame are placed 6 ft. apart, the gas-flame being of 4 candle-power: where must a screen be placed, on the line joining the candle and gas-flame, so that it may be equally illuminated by each of them?

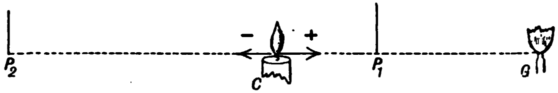

Fig. 2.

Let x denote the distance of the screen from the candle; its distance from the gas-flame will then be $6 - x$. If the screen is to receive equal illumination from each of the sources of light we must have

$$1/x^2 = 4/(6-x)^2,$$

$$\text{i.e. } 6 - x = \pm 2x.$$

and $$\therefore x = +2 \text{ or } -6.$$

Corresponding to these two values of x there are two positions (P_1 and P_2) of the screen which satisfy the conditions of the problem. The first, P_1, is 2 ft. to the *right* of the candle, *i.e.* between the candle and the gas-flame; the second, P_2, is 6 ft. to the *left* of the candle.

Both solutions can easily be verified.

3. In determining the illuminating power of a gas-flame by Bunsen's photometer, the distance from the gas-flame to the grease spot was 84 cm., and from this to the standard candle 40 cm.: what was the candle-power of the gas-flame?

4. A candle is placed at a distance of 1 ft. from one side of a card-board screen, and a lamp of 9 candle-power is placed at a distance of 12 ft. on the other side. Compare the illumination on the two sides of the screen.

5. A Carcel lamp of 9 candle-power is placed at a distance of 4 yards from a standard candle: determine, as in Example 2, the two positions in which a screen may be placed so as to receive equal amounts of light from the lamp and the candle.

6. What do you understand by the terms "real image," "virtual image"? Draw a figure showing how an image is formed in a plane mirror, and prove that the object and its image are equally distant from the mirror.

7. A candle is placed in any position between two plane mirrors at right angles; show that its two secondary images (produced by reflection from both mirrors) will exactly coincide.

8. A luminous object is placed between two plane mirrors inclined at 60°: find the number of images, and show that they all lie on a circle.

9. An object is placed between two mirrors inclined at 45° : show by a figure how an observer could see an image after four successive reflections.

10. A candle is placed between two parallel mirrors; draw a sketch showing the path of a ray of light which proceeds from the candle, and, after undergoing two reflections at one mirror, and three at the other, enters the eye of an observer looking toward the mirrors.

11. Two plane mirrors, A and B, are placed vertically upon a horizontal table. A ray of light PB falls upon the mirror B, and is reflected to the mirror A; show that the ray AQ reflected from the latter makes with PB an angle which is double of the angle between the mirrors.

12. If the mirrors of a kaleidoscope are placed at an angle of 45°, how many images will there be of an object (1) placed close to one of the mirrors, (2) placed midway between the mirrors?

13. AB and AC are two plane mirrors inclined at an angle of 15°, and P is a point in AB. At what angle must a ray of light from P be incident upon AC, in order that after three reflections it may be parallel to AB?

14. An object is placed in front of a convex spherical mirror: find by a geometrical construction the position and nature of the image; and show that the sizes of image and object are as their distances from the centre of the mirror.

15. Assuming that in the case of a spherical mirror the sizes of image and object are proportional to their distances from the centre of the mirror, show that they are also proportional to their distances from the mirror itself.

16. An object is placed at a distance p from a concave mirror of focal length f: prove that the image is formed at a distance p', which is given by the equation,

$$1/f = 1/p + 1/p'.$$

Rules for Optical Calculations.—In order to avoid confusion as to plus and minus signs, or positive and negative focal distances, beginners will find it well to follow strictly some such rules as the following, and to employ the equations for conjugate positions always in the same form.

I. All distances are to be reckoned from the mirror or lens: if measured towards the right hand they are to be considered positive; if towards the left hand negative. It is convenient to suppose that the luminous object, or source of light, is always placed on the right hand of the mirror or lens: its distance will thus always be positive.

II. The formula

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{p'} \quad . \quad .. \quad . \quad . \quad (1)$$

holds good for all mirrors, both concave and convex, f denoting the focal length, p the distance of the object, and p' the distance of the image, reckoned from the mirror itself.

For all lenses, convex and concave, the equation for conjugate points is

$$\frac{1}{f}=\frac{1}{p'}-\frac{1}{p} \quad . \quad . \quad . \quad . \quad (2)$$

the letters denoting the same quantities as before, *and being always supposed to carry their own signs.*

III. The focal length of a concave mirror is positive; of a convex mirror negative. The focal length of a convex (convergent) lens is negative; of a concave (divergent) lens positive, *i.e.*,

$$\text{for}\left\{\begin{array}{l}\text{concave mirrors}\\ \text{concave lenses}\end{array}\right\} f \text{ is positive } (+),$$

$$\text{for}\left\{\begin{array}{l}\text{convex mirrors}\\ \text{convex lenses}\end{array}\right\} f \text{ is negative } (-).$$

When any two of the three quantities f, p, and p' are given, the third can be found by means of equation (1) or (2). If f and p are given, the numerical value of p' indicates the distance from the mirror or lens at which the image is formed: if p' is positive, the image is on the same side as the object (towards the right hand); but if it carries the minus sign, the image is formed on the opposite side, *i.e.* on the left hand of the mirror or lens. When the distances of object and image (p and p') are given, the nature of the mirror or lens is indicated by the sign of f: thus the mirror or lens is concave if f is positive, and is convex if f is negative.

17. An object is placed at a distance of 10 cm. in front of a convex mirror of 30 cm. focal length; where will the image be formed?

Here $p = 10$, and $f = -30$. Substituting these values in equation (1) we have

$$-1/30 = 1/10 + 1/p',$$

or

$$1/p' = -1/30 - 1/10 = -4/30,$$

and $\therefore p' = -7{\cdot}5$; *i.e.* the image is formed 7·5 cm. *behind* the mirror.

18. A real image produced by a concave mirror is found to be three times the size[1] of the object: if the mirror is one foot from the object, what is its focal length?

Since the image is three times the size of the object, its distance from the mirror must be three times that of the object; *i.e.* $p' = 3p = 3$ ft.

Thus $1/f = 1 + 1/3 = 4/3$, and $f = \frac{3}{4}$ ft. $= 9$ inches.

[It should be noticed that when the sizes of the image (I) and object (O) are given, the relation

$$I : O = p' : p$$

only gives the *numerical* value of p'; so that in this example if the image had been virtual we should have had $p' = -3$.]

19. A luminous point is 24 cm. in front of a concave mirror of 6 cm. focal length: where is the image formed? If the point moves through a small distance d away from the mirror, through what distance will the image move?

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[1] When not otherwise stated, it may be assumed that the word "size" refers to *linear* dimensions.

(1.) The value of p' is given by the equation

$$1/6 = 1/24 + 1/p',$$

from which we have $p' = 8$. The image is therefore formed 8 cm. in front of the mirror.

(2.) The new value of p is $24 + d$, and the corresponding value of p' is given by

$$1/6 = 1/(24 + d) + 1/p',$$
$$\therefore p' = 6(24 + d)/(18 + d).$$

This is less than 8, and the distance through which the image moves is

$$8 - 6(24 + d)/(18 + d) = 2d/(18 + d).$$

20. You are required to throw upon a wall an image of a gas-flame which stands 8 ft. from the wall, and the image is to be three times the size of the flame: what sort of mirror would you choose, and where would you hold it?

Suppose the mirror to be placed x feet from the object on the side farther from the wall; it will then be $(8 + x)$ ft. from the wall, and, since the image is to be three times the size of the object, we must have $p' = 3p$, or

$$8 + x = 3 \times x, \text{ and } \therefore x = 4.$$

Thus $p = 4, p' = 8 + 4 = 12,$

and $1/f = 1/4 + 1/12 = 1/3.$

The mirror required is a concave mirror of 3 ft. focal length, and it must be held 4 ft. from the object.

21. A candle-flame 1 in. long is 18 in. in front of a concave mirror whose focal length is 15 in.: find the position and size of the image.

22. Prove that when an object is placed midway between a concave mirror and its principal focus, the image is twice as large as the object. What is the nature of the image?

23. A luminous point is 60 cm. in front of a mirror, and its image is found to be 20 cm. from the mirror, and on the same side: find the nature and focal length of the mirror.

24. A gas-jet is placed on the principal axis of a spherical mirror and 1 ft. in front of it. A real and inverted image is produced on a screen held in front of the mirror, but at a greater distance than the candle. If the image is twice as long as the flame, what is the focal length of the mirror?

25. An object is placed 5 in. from a concave mirror of 6 in. focal length: where is the image produced, and what is the magnification?

26. It is desired to throw upon a wall an image of an object magnified twelve times, the object being 11 ft. from the wall: find the focal length of a concave mirror that may be used for the purpose, and state where it must be placed.

27. An object 1 inch in length is held 6 in. in front of a convex mirror whose radius of curvature is 2 ft.: find the nature, position, and magnitude of the image.

Where will the image be produced when the object is held 2 ft. in front of the same mirror, and what will be its size?

28. An image produced by a concave mirror is found to be twice the size of the object: if the focal length of the mirror is 1 ft., where are the object and image situated? What would be the relation between the sizes if their positions were reversed?

29. Prove that when an object is at a distance $2f/3$ from a concave mirror of focal length f, the image produced is erect and virtual, and magnified three times.

30. An object is placed at a distance $3f/2$ in front of a concave mirror of focal length f: what is the size of the image? Is there any other position of the object which will produce the same degree of magnification?

31. At what distance from a concave mirror (of focal length f) must an object be situated in order that the image may be half the size of the object?

32. An object is held in front of a convex mirror at a distance equal to its focal length: what is the size of the image?

33. An image produced by a convex mirror is $1/n$th the size of the object: prove that the latter must be at a distance $(n-1)f$ from the mirror.

34. Enunciate the laws of refraction of light, and explain what is meant by the index of refraction of a substance.

35. A ray of light is incident upon the surface of still water, and the index of refraction for air and water is $\frac{4}{3}$. Show how the path of the ray in water can be found when its path in air is known: and discuss any special cases which may occur when the direction of the ray is reversed, *i.e.* when it travels upwards towards the surface of the water.

36. Describe an experiment to show total internal reflection, and point out the conditions under which it occurs. What is the *critical angle* for two media?

37. Given the indices of refraction from a medium A to another B, and from B to a third C: show how to determine the index of refraction from A to C.

38. The refractive index for air and water is $4/3$, and for air and glass $3/2$: find the index from glass to water and from water to air.

39. A ray of light passes from one medium into a second, the angle of incidence being 60° and the angle of refraction 30°: show that the index of refraction is $\sqrt{3}$.

40. Find the index of refraction when the angles of incidence and refraction are 45° and 30° respectively.

41. The critical angle for a certain medium is 45°: what is its refractive index?

42. A microscope is placed vertically above a short glass tube cemented to a flat piece of glass (so as to form a tube open at the top), and is focussed upon a mark on the glass slip. A layer of liquid of depth h is poured into the tube, and it is now found that the image of the mark is displaced through a distance d which is determined by refocussing the microscope. Prove that the refractive index of the liquid is $\mu = h/(h-d)$.

43. In an experiment made by the above method water was poured into a depth of 4·6 cm., and the resulting displacement of the image was 1·15 cm. Calculate the refractive index of water.

44. Calculate the refractive index of a glass prism for sodium light from the following observations:—

Refracting angle of prism	45° 4′.
Minimum deviation	26° 40′

It can be shown that when a ray of light passes through a prism of angle A placed in the position of minimum deviation, the index of refraction of the prism is given by

$$\mu = \sin\frac{A+D}{2} \Big/ \sin\frac{A}{2},$$

where D is the angle of deviation.

$$\begin{aligned} \text{Here}\qquad A &= 45^\circ\ 4' \\ \underline{D} &\underline{= 26^\circ\ 40'} \\ A+D &= 71^\circ\ 44' \\ \text{and}\qquad (A+D)/2 &= 35^\circ\ 52'. \end{aligned}$$

From the table of natural sines we find that

$$\sin 35^\circ = 0{\cdot}574, \text{ and } \sin 36^\circ = 0{\cdot}588.$$

Assuming that the rate of change of the sine is proportional to that of the angle within these limits, we have

$$\sin 35^\circ\ 52' = 0{\cdot}586 \text{ approximately.}$$

In the same way we obtain 0·384 as the approximate value of sin 22° 32′. Thus

$$\mu = 0{\cdot}586/0{\cdot}384 = 1{\cdot}526.$$

45. Find the refractive index of the same prism for lithium light, the minimum deviation produced being 26° 30′.

46. The minimum deviation produced by a hollow prism filled with a certain liquid is 30°: if the refracting angle of the prism is 60°, what is the index of refraction of the liquid?

47. A prism is to be made of crown glass, the refractive index of which is known to be 1·526, and it is required to produce a minimum deviation of 17° 20′: to what angle must it be ground?

48. A ray of light is incident almost perpendicularly upon a prism of angle a and refractive index μ: show that if a is small the deviation is given by

$$\delta = (\mu - 1)a.$$

49. The refractive index of rock-salt is 1·54: what deviation would be produced by a rock-salt prism of 1° 30′ angle? and what should be the angle of a rock-salt prism which is required to produce a deviation of 48′?

50. Taking the usual values for the refractive indices of water and glass, prove that the deviations produced by thin prisms of water and glass are one-third and one-half of the angles of the prisms respectively.

Note.—The refractive index for water and air is 4/3 ($1\cdot\dot{3}$), and for glass and air 3/2 (or 1·5).

51. Find the angle of a water prism which will produce the same deviation as that given by a glass prism of 2° angle.

52. In order to determine the refractive index of a double convex lens, its focal length and the radii of curvature of its faces were measured, and were found to be

$$f = 30{\cdot}6 \text{ cm.}, \ r_1 = 30{\cdot}4 \text{ cm.}, \ r_2 = 34{\cdot}5 \text{ cm.}$$

What was the index of refraction of the glass?

It can be proved[1] that the focal length of a lens, in terms of its refractive index and the radii of curvature of its surfaces, is given by the formula $1/f=(\mu-1)(1/r_1-1/r_2)$.

In applying this we must remember that the radius of curvature of the first surface is negative, or $r_1=-30{\cdot}4$; and since the lens is convex, its focal length is also negative.

Thus $\quad -1/30{\cdot}6=(\mu-1)(-1/30{\cdot}4-1/34{\cdot}5)$,

or $\quad \mu-1=30{\cdot}4\times 34{\cdot}5/30{\cdot}6\times 64{\cdot}9=0{\cdot}528$,

and $\quad \therefore \mu=1{\cdot}528$.

53. Taking the usual values for the refractive indices of water and glass, prove that the focal length of a glass lens when immersed in water is four times its focal length in air.

54. Prove that the focal length of a plano-concave glass lens is equal to twice the radius of the concave surface.

55. A double convex lens is to be made of glass of refractive index 1·5, and the radius of curvature of one of its faces is 20 cm. If the lens is to have a focal length of 30 cm., what must be the radius of curvature of the other face?

56. The radii of curvature of the two faces of an equiconvex lens are each equal to 46·5 cm., and it is made of glass of refractive index 1·532: show that its focal length is 43·7 cm.

57. Explain what is meant by the optical centre of a lens, and prove that the optical centre of a plano-convex or plano-concave lens lies on the curved surface.

58. A candle stands at a distance of 3 ft. from a wall: in what position must a convex lens of 8 in. focal length be placed between them so as to produce upon the wall a distinct image of the candle?

Let p denote the distance (in inches) of the candle from the lens; then $36-p$ will be the distance of the screen from the lens *irrespective of sign.* Remembering that p' is negative, we have $p' = p - 36$, and now substituting in equation 2 (p. 155) we have

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[1] Aldis, *Geometrical Optics*, Art. 67.

$$-1/8 = 1/(p-36) - 1/p,$$
$$\therefore\ p^2 - 36p = 8p - 8p - 288,$$

or
$$p^2 - 36p + 288 = 0.$$

This may be written in the form

$$(p-24)(p-12) = 0,$$

and
$$\therefore\ p = 24 \text{ or } 12.$$

A distinct image will therefore be produced when the candle is placed either 1 ft. or 2 ft. from the lens.

59. If an object at a distance of 3 in. from a convex lens has its image magnified three times, what is the focal length of the lens?

There are two solutions to this problem, for the image may be either real or virtual. In the first case it is formed on the other side of the lens; in the second case on the same side as the object.

In both cases, since the image is three times as large as the object, its distance from the lens must be three times that of the object; but this only gives us the *numerical* value of p' in terms of p.

(1) Image real—

$$p' \text{ is negative and} = -3p \text{ or } -9 \text{ in.}$$

Thus
$$1/f = 1/p' - 1/p,$$
$$= -1/9 - 1/3 = -4/9,$$

and the focal length of the lens is $-2\frac{1}{4}$ inches.

(2) Image virtual—

$$p' \text{ is positive and} = 3p = +9 \text{ in.}$$
$$1/f = 1/9 - 1/3 = -2/9,$$

so that the focal length in this case is $-4\frac{1}{2}$ inches.

60. Explain the difference between real and virtual images, and give examples of each. If you had a convergent lens of 1 ft. focal length, where would you place an object so as to produce by means of the lens (1) a real and diminished image, (2) an erect and virtual image? Give sketches showing how the image is produced in each case.

61. Rays of light diverging from a point 6 in. before a lens are brought to a focus 18 in. behind it: what is the focal length of the lens?

62. An object is placed at a distance of 60 cm. from a convex lens of 15 cm. focal length: where is the image formed? Compare its size with that of the object.

63. An object whose length is 5 cm. is placed at a distance of 12 cm. from a convex lens of 8 cm. focal length: what is the length of the image?

64. A candle is placed at a distance of 10 ft. from a wall, and it is found that when a convex lens is held midway between the candle and the wall a distinct image is produced upon the latter. Find the focal length of the lens and the relative sizes of the object and image.

65. A coin half an inch in diameter is held on the axis of a convergent lens, and 1 ft. in front of it: if the focal length of the lens is 8 in., find the position and magnitude of the image.

66. Draw figures, approximately to scale, showing the paths of the rays of light, and the positions of the images formed when a luminous object is placed at a distance of (1) 1 inch, (2) 6 inches from a convergent lens of 2 in. focal length.

67. An object is placed 8 in. from a convex lens, and its image is formed 24 in. from the lens on the other side. If the object were placed 4 in. from the lens, where would the image be?

68. The distance of an object from a convergent lens is double the focal length of the lens: prove that the image and object are of the same size.

69. A candle stands at a distance of 2 metres from a wall, and it is found that when a lens is held half a metre from the candle a distinct image is produced upon the wall: find the focal length of the lens, and also state the relative sizes of image and object.

70. A lens of 9 in. focal length is to be used for the purpose of producing an inverted image of an object magnified three times: where must each be situated?

71. A convex lens is held 5 ft. in front of a wall, and it is found that there is one position in which an object can be held in front of the lens such that an inverted image six times as large as it is thrown upon the wall. Determine this position, and also find the focal length of the lens.

72. At what distance from a convex lens must an object be placed so that the image may be half the size of the object?

73. You are provided with a convex lens of 18 in. focal length, and are required to place an object in such a position that its image will be magnified three times: find the positions which will give (1) a real, and (2) a virtual image of the required size.

74. In order to find the focal length of a concave lens, it was blackened, with the exception of a circle 4 cm. in diameter at its centre. A beam of sunlight was allowed to pass through this, when it was found that an illuminated circle of 20 cm. diameter was formed on a screen held 64 cm. behind the lens and parallel to it. What was the focal length of the lens?

75. A convex lens produces a real image n times as large as the object: prove that the latter must be at a distance $(n+1)f/n$ from the lens.

76. A glass scale, 4 cm. long, was held in front of a convergent lens, and on holding a screen 90 cm. behind the lens, an image of the scale, 20 cm. in length, was produced upon the screen: prove that the lens had a focal length of 15 cm.

77. Show how to find the focal length (F) of a combination obtained by placing two thin lenses of focal lengths f_1 and f_2 in contact. Prove that for any number of such lenses placed in contact $1/F = \Sigma(1/f)$.

78. What is the focal length of a lens which is equivalent to two thin convergent lenses of focal lengths 15 cm. and 30 cm. placed in contact?

79. A concave lens of 8 cm. focal length is combined with a convex lens of 6 cm. focal length: what is the focal length of the combination?

80. A convex lens of focal length 16 cm. was placed in contact with a concave lens, and the focal length of the combination was found to be 48 cm. Calculate the focal length of the concave lens.

81. A candle is held 1 foot in front of a convex lens, and a distinct image of the flame is formed on a screen placed 4 inches behind it. A concave lens is now placed in contact with it, and it is found that the screen has to be moved 8 inches farther off in order to receive the image. What is the focal length of the concave lens?

82. Explain the action of a condensing lens when used as a magnifying glass. Give a sketch showing how the image is produced, and prove that the magnifying power is approximately equal to Δ/f, where Δ is the distance of most distinct vision.

83. Describe the action of the eye, considered as an optical instrument, and explain the causes of abnormal vision. Will the magnifying effect of a given reading-lens be greater when used by a long-sighted or a short-sighted person?

84. A person whose distance of most distinct vision is 20 cm. uses a lens of 5 cm. focal length as a reading-glass: at what distance from a book must he hold it? Also what will be its magnifying power?

85. A long-sighted person can only see distinctly objects which are at a distance of 48 cm. or more: by how much will he increase his range of distinct vision if he uses convex spectacles of 32 cm. focal length?

86. A short-sighted man can read printed matter distinctly when it is held at 15 cm. from his eyes: find the focal length of the glasses which he must use if he wishes to read with ease a book at a distance of 60 cm.

87. A convex lens produces an image of a candle-flame upon a screen whose distance from the candle is l; the lens is displaced through a distance d, when it is found that a distinct image is again produced upon the screen. Show that the focal length of the lens is $(l^2 - d^2)/4l$.

88. Prove that the size of the object in the last question is a geometrical mean between the sizes of the two images produced.

89. In an experiment made according to the method of Ex. 82, the distance between the candle and screen was 255 cm. and the lens had to be shifted through a distance of 73·7 cm. What was its focal length?

90. Calculate the mean value of the focal length of a convex lens which gave the following results by the method of displacement:—

Exp. 1 . . . $l = 85$ cm. $d = 38{\cdot}7$ cm.
" 2 . . . $l = 80$ " $d = 33{\cdot}0$ "
" 3 . . . $l = 70$ " $d = 14{\cdot}5$ "

EXAMINATION QUESTIONS.

91. Describe a method of measuring the velocity of light (*a*) in air, (*b*) in glass, its velocity in air being known. In using Foucault's method it was observed that when the mirror was turning 257 times per second the displacement of the image was ·113 metre, the distance between the slit and the moving mirror being 8·58 metres, and between the two mirrors 605 metres. Show that the velocity of light is 296,000,000 metres per second. Camb. Schol. 1883.

92. State the principles on which the illuminating powers of two sources of light are compared. The distance between two incandescent lamps, of 16 and 25 candle-power respectively, is 6 feet. Show that there are two positions, on the line joining the lamps, in which a screen may be placed so as to receive equal illumination from each lamp, and determine these positions.

Prel. Sc. 1887.

93. On a moonlight night, when the surface of the sea is covered with small ripples, instead of an image of the moon being seen in the sea, a long band of light is observed on the surface of the sea extending towards the point which is vertically beneath the moon. Account for this phenomenon in accordance with the laws of reflection, illustrating your explanation by a figure.

Matric. 1882.

94. What is the index of refraction of a transparent substance?

A plate of glass 6 inches thick with a refractive index of 1·5 is placed 2 inches above a luminous object. Make a careful full-sized drawing showing the path of a small conical pencil of light through the plate, the axis of the pencil being normal (or perpendicular) to the surface of the glass, and show where the image of the object will appear to an eye placed on the other side of the plate. Matric. 1884.

95. Define the term "the refractive index of a transparent medium," and give an account of experiments by which that of a liquid may be measured.

The refractive index of water is 1·33, and the velocity of light in air is 300,000,000 metres per second. Find its value in water, stating the experimental grounds there are for your answer. Ind. C. S. 1885.

96. Under what circumstances is total internal reflection possible? A ray of light passing through a certain medium meets the surface, separating the medium from air at an angle of 45°, and is just not refracted. What is the refractive index of the medium? Matric. 1887.

97. What is meant by saying that the refractive indices of glass and of water are 1·5 and 1·33 respectively? Show for which of these substances the *critical angle*, or *limiting angle of refraction*, is the greater.
Matric. 1885.

98. An image of a candle-flame eight times as broad as the flame itself is to be thrown, by means of a convex lens, on a wall at a distance of 12 feet from the candle. What will be the focal length of the lens required, and where must it be placed? Matric. 1885.

99. How is the focal length of a convex lens best determined without the aid of sunlight?

An object is placed 8 inches from the centre of a convex lens, and its image is found 24 inches from the centre on the other side of the lens. If the object were placed 4 inches from the centre of the lens where would the image be? Matric. 1886.

100. An object 3 inches in height is placed at a distance of 6 feet from a lens, and a real image is formed at a distance of 3 feet from the lens. The object is then placed 1 foot from the lens. Where, and of what height, will the image be? Matric. 1887.

101. A goldfish globe of 6 inches radius is filled with water. Determine the apparent position of a point inside the globe, 4 inches from its surface, when seen by an eye outside looking along a radius of the globe.
Int. Sc. 1884.

102. A small direct pencil of rays from a luminous point enters a refracting medium bounded by a spherical surface. Determine the image of the point.

Given a double concave lens of 5 cm. thickness, the radii of curvature of its faces being 15 and 20 cm. respectively. Find the position of the image of a point in the axis 24 cm. from the nearer face, aberration being neglected. Int. Sc. Honours 1884.

103. Trace the position of the image of a bright point formed by a lens consisting of a sphere of glass,

of radius 2 inches and refractive index 1·5, when the point moves from an infinite distance up to the sphere.
Int. Sc. Honours 1885.

104. Explain fully how you would determine experimentally the index of refraction of a plano-convex lens for sodium light. Int. Sc. Honours 1883.

105. A luminous point is placed in the axis of a glass hemisphere, for which $\mu = 3/2$, at a distance of a foot from the plane surface; if the radius of the hemisphere be 9 inches, show that the rays after passing through it will be parallel. Camb. Schol. 1886.

106. A small air-bubble in a sphere of glass 4 inches in diameter appears, when looked at so that the bubble and the centre of the sphere are in a line with the eye, to be 1 inch from the surface. What is its true distance? ($\mu = 1 \cdot 5$.) Int. Sc. 1887.

107. A convex lens of 6 inches focal length is used to read the graduations of a scale, and is placed so as to magnify them three times; show how to find at what distance from the scale it is held, the eye being close up to the lens. Owens Coll. 1886.

108. A pair of spectacles is made of two similar lenses, each having two convex surfaces of 10 and 20 inches radius respectively, and a refractive index 1·5. A person seeing through them finds that the nearest point to which he can focus is 1 foot away from the glasses. What is his nearest point of distinct vision?
Camb. M.B. 1885.

109. How is a spectrum obtained by diffraction? How does such a spectrum differ from a prismatic spectrum?

If a grating with 100 lines to the millimetre is placed in front of a slit illuminated with monochromatic light, and the angular distances of the 1st and 2d images are found to be 2° 18′ and 4° 35′ from the central image, what is the wave length of the light?

$$\sin 2^\circ\ 18' = \cdot 0401.$$
$$\sin 4^\circ\ 35' = \cdot 0799.$$

Balliol Coll. 1881.

110. Light from two exactly equal and similar small sources very close together falls on a screen. Account for the bands seen, explaining the difference in the appearance, according as the light is white or of some definite refrangibility.

The distance between the two sources of light is ·184 cm., and the distance between the sources and the screen is 112 cm.; a series of bright and dark bands at a distance of ·036 cm. apart is observed on the screen. Find the wave length of the light used. Ind. C. S. 1885.

111. The minimum deviation of a ray of light produced by passing through a prism of angle 60° 6′ 20″ is 42° 40′ 20″. Show how to use these results to determine the refractive index of the glass prism, and find it, having given—

$$\text{L}\sin 51^\circ\ 24' = 9\cdot 89294,\ \text{L}\sin 30^\circ\ 4' = 9\cdot 69984,$$
$$\text{L}\sin 51^\circ\ 23' = 9\cdot 89284,\ \text{L}\sin 30^\circ\ 3' = 9\cdot 69963.$$
$$\log 1\cdot 5610 = \cdot 19340,\ \log 1\cdot 5600 = \cdot 19312.$$

Int. Sc. Honours 1886.

CHAPTER VII

SOUND

Velocity of Sound.—Newton proved that the velocity of sound in any medium is given by the equation $V = \sqrt{E/D}$, E denoting the elasticity and D the density of the medium.

The *elasticity* of a fluid is defined as being the ratio of any small increase of pressure to the proportional decrement of volume thereby produced. It can be shown that the elasticity of a perfect gas is equal to its pressure, provided that its temperature remains constant during the compression.

A geometrical proof of this important proposition is given in Maxwell's *Theory of Heat*. It may also be proved as follows :—

Let V be the volume of a given mass of gas under the pressure P. Now suppose the pressure to increase by a small amount p, and let v be the decrement of volume thereby produced: the pressure is now $P+p$, and the volume $V-v$. If the gas obeys Boyle's law, the product of these two quantities is equal to the product of the original pressure and volume, or

$$\begin{aligned} PV &= (P+p)(V-v), \\ &= PV + Vp - Pv - pv. \end{aligned}$$

Since both the quantities p and v are small, their product may be neglected (p. 17); thus

$$Vp = Pv.$$

Now the *proportional decrement of volume* (or decrement per unit volume) is $\frac{v}{V}$, so that the elasticity (by definition) is $p\Big/\frac{v}{V}$ or Vp/v. But by the last equation

$$Vp/v = P,$$

and therefore the elasticity is equal to the pressure.

We have seen (p. 70) that if the barometer stands at h cm., the atmospheric pressure is $\Pi = h\rho g$ dynes per sq. cm. This gives us for the velocity of sound in air at 0°

$$V = \sqrt{h\rho g/D},$$

or, at the normal pressure,

$$V = \sqrt{1{,}013{,}226/0{\cdot}001{,}293},$$
$$= 279{,}933 \text{ cm. per sec.},$$

whereas the velocity found by experiment is 332 metres per sec. This discrepancy between the calculated and observed values remained unexplained until Laplace's time. He pointed out that the compression produced by a sound-wave takes place so rapidly that any heat which is developed by it cannot be conducted away: the elasticity cannot therefore be calculated on the supposition that the temperature remains constant. If we assume that no heat is allowed to escape, it can be proved that the elasticity is equal to $\gamma\Pi$, Π being the pressure and γ the ratio between the specific heat of air at constant pressure and its specific heat at constant volume. Introducing this correction (known as Laplace's correction) we have

$$V = \sqrt{\gamma h\rho g/D},$$

or taking $\gamma = 1{\cdot}4$,

$$V = \sqrt{1{\cdot}4 \times 1013226/0{\cdot}0001293},$$
$$= 331{,}221 \text{ cm. per sec.},$$

which is almost identical with the velocity found by experiment.

1. Explain, on general principles, why the velocity of sound in air increases with the temperature, but is

independent of the pressure; and calculate its velocity at 15°.

Note.—The velocity of sound in air at 0° is 332 metres per second.

2. Find the temperature at which the velocity of sound in air is 350 metres per second.

3. An observer sets his watch by the sound of a signal-gun fired at a station 1500 metres off: find, to a hundredth of a second, the error due to distance, the temperature being 15°.

4. Show that the left and right hand members of the equation $V = \sqrt{E/D}$ are of the same dimensions.

5. A tuning-fork is held over a tall glass jar, into which water is gradually poured until the maximum reinforcement of the sound is produced. This is found to be the case when the length of the column of air is 64·8 cm. What is the vibration number of the fork?

6. Calculate the velocity of sound in hydrogen gas, assuming its velocity in air, and having also given that 1 litre of hydrogen = 0·0896 gm., and 1 litre of air = 1·293 gm.

7. Find the length of a closed organ-pipe, which when blown will give the note *c* (256 vibrations per second).

8. Calculate the velocity of sound in water at 10°, its coefficient of elasticity at this temperature being $2 \cdot 1 \times 10^{10}$.

The velocity of sound in liquids is given by the same expression $\sqrt{E/D}$, E denoting the coefficient of elasticity (or the reciprocal of the coefficient of compressibility) and D the density.

At 10° the density of water is sensibly equal to unity, and the required velocity is

$$V = \sqrt{2 \cdot 1} \times 10^5 = 144{,}900 \text{ cm. per sec.}$$

9. By experiments made in the Lake of Geneva, Colladon and Sturm found that sound travelled in water at 8°·1 with a velocity of 1435 metres per sec. What value does this give for the elasticity of water?

10. It is found that a force equal to the weight of 3000 lbs. is required to elongate a bar of iron 1 sq. inch in section by 1/10,000 of its original length: calculate from this the velocity of sound in iron. [1 cub. ft. of iron = 480 lbs.]

In calculating the velocity of sound in solids, E is to be taken as denoting Young's modulus of elasticity, the value of which for iron, *in poundals per square foot*,[1] is $10{,}000 \times 3000 \times 32 \times 12^2$.

The density of iron, in lbs. per cub. ft., is given as 480. Thus

$$V = \sqrt{E/D},$$
$$= \sqrt{9{\cdot}6 \times 10^8 \times 12^2/480} = 17{,}000 \text{ ft. per sec.}$$

11. Calculate the value of Young's modulus for steel, having given that its density is 7·8, and that sound travels in it with a velocity of 5200 metres per second.

12. What will be the pitch of the note emitted by a wire 50 cm. in length, when stretched by a weight of 25 kilogrammes, if 2 metres of the wire are found to weigh 4·79 gm.?

It can be proved that a sound-wave travels along a stretched wire or string with a velocity $v = \sqrt{F/M}$, where F is the stretching force, and M is the mass of the wire per unit of length.[2]

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[1] The modulus of elasticity (Young's modulus) is defined as follows:—Suppose a bar or wire of length L and cross-section σ to be stretched by a force F, and let l denote the elongation produced; then the modulus of elasticity for the material of which it is composed is $E = LF/l\sigma$. The modulus is frequently expressed in lbs. per sq. in., or kilogrammes per sq. cm.; in such cases it must be multiplied by g, and care should be taken to use the same units consistently in the calculation, *e.g.* if we wish to find the velocity in centimetres per sec., the modulus must be expressed in dynes per sq. cm. and the density in grammes per c.c.

[2] If the wire be of density d, $M = \pi r^2 d$, so that if we denote by t the tension of the wire per unit of sectional area, $t = F/\pi r^2$, and therefore $v = \sqrt{t/d}$. The complete expression for the number of vibrations per second produced by a wire of density d stretched by a weight P is

$$n = \frac{1}{2lr}\sqrt{\frac{Pg}{\pi d}}.$$

If λ be the wave-length of the note, and n the number of vibrations per second producing it, then $v = n\lambda$: also the length l of the stretched wire must be an exact multiple of $\lambda/2$. When the string is sounding its fundamental note, l is equal to $\lambda/2$, and

$$n = \frac{v}{\lambda} = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{F}{M}}.$$

The stretching force in the present example is the weight of 25 kgm. $= 25{,}000 \times 981$ dynes; and $M = 4{\cdot}79/200 = 0{\cdot}02395$ gm. Thus $n = \frac{1}{100}\sqrt{\frac{25{,}000 \times 981}{0{\cdot}02395}} = 320$; and if we take the vibration number of *c* as 256, the note emitted will be *e*, for $320/256 = 5/4$, and this interval[1] is a major third.

13. Two similar wires of the same length are stretched —the one by a weight of 4 lbs., and the other by a weight of 9 lbs. What is the interval between the notes which they produce?

14. A stretched string 3 feet long gives the note *c* when vibrating transversely: what note will be given by a string 1 foot long stretched by the same weight and made of the same material, but of one-quarter the thickness?

15. A vibrating string is found to give the note *f* when stretched by a weight of 16 lbs. What weight must be used to give the note *a*? and what additional weight will give *c'*?

16. A wire 50 cm. in length and of mass 80 gm. is stretched so that it vibrates eighty times per sec.: find the stretching force in dynes.

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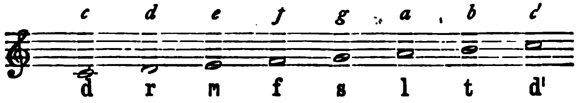
[1] The intervals of the diatonic scale (from *c* to *c'*), and the vibration numbers of the notes, taking $c = 256$, are as follows:—



$1 : \frac{9}{8} : \frac{5}{4} : \frac{4}{3} : \frac{3}{2} : \frac{5}{3} : \frac{15}{8} : 2$

$256 : 288 : 320 : 341{\cdot}\dot{3} : 384 : 426{\cdot}\dot{6} : 480 : 512$

17. A copper wire (density 8·8) 1 metre in length and 1·8 mm. in diameter is stretched by a weight of 20 kilogrammes. Calculate the number of vibrations which it makes per second when sounding its fundamental note.

18. The *e* string of a violin is tuned so as to vibrate 640 times per second; the vibrating portion has a mass of $\frac{1}{8}$ of a gramme and a length of 33 cm. What is the stretching force?

19. An observer listens to a whistle sounded on a railway train as it comes toward him, and the pitch of the whistle appears to be *f*♯, but just as the train passes him the pitch falls to *f*. Show the speed of the train may be deduced from these observations.

The note actually emitted by the whistle is *f*, but while the train is approaching the observer the apparent pitch is heightened, because a larger number of sound-waves enter his ear per second (Döppler's principle).

Suppose the train to be at a distance d, and moving with a velocity of v ft. per sec. toward the observer; also let n be the vibration number of the note *f*. Consider what happens while the engine moves through v ft. (*i.e.* during 1 sec.) Assuming that the velocity of sound in air is 1100 ft. per sec., the first vibration, produced at a distance d, will reach the observer in a time $d/1100$. When the nth vibration is produced the train is v ft. nearer, and this last vibration will reach him in a time $d/1100 - v/1100$. Thus his ear receives n vibrations in $(1 - v/1100)$ sec., or $n/(1 - v/1100)$ per sec., which is therefore the vibration number of the note *f*♯. The interval between this note and *f* is a minor semitone, and is equal to 25/24, so that

$$\frac{n}{1 - v/1100} = \frac{25n}{24},$$

and $$v = 1100/25 = 44.$$

Thus the speed of the train is 44 ft. per sec., or 30 miles per hour.

20. An observer listening to the whistle of an engine which is approaching him at the rate of 45 ft. per sec., notices that the pitch of the note which he hears is the same as that of a tuning-fork which makes 458 vibrations per sec. What is the actual pitch of the whistle? (Velocity of sound in air = 1100 ft. per sec.)

21. Give a graphical illustration of the manner in which "beats" are produced, and show that the number of beats per second can be calculated from the vibration numbers of the two notes producing them.

Two open pipes are sounded together, each note consisting of its first two harmonics, together with the fundamental. One note has 256 vibrations per second, the other 170. Show that two of the harmonics will produce beats at the rate of two per second.

22. A smoked-glass plate is held vertically in front of a vibrating fork provided with a style, and the plate is allowed to fall freely under the action of gravity, so that the style traces a wavy line upon it. Prove that if the number of waves marked in a distance d (starting from rest) be n, then the vibration-number of the fork is $n/\sqrt{2d/g}$.

23. In an experiment made according to this method, it was found that in a distance of 10·9 cm. (measured from the position of rest) 68½ waves were included. Find the vibration-number of the fork, taking $g = 980$.

EXAMINATION QUESTIONS.

24. State clearly what is meant by the elasticity of water. If the elasticity of water be 2×10^{10} C.G.S. units, calculate the velocity of sound in it.

Int. Sc. Honours 1883.

25. Describe the mode of transmission of a sound-wave in air. The velocity of sound in any gas is numerically equal to the square root of the ratio of the numerical value of the elasticity of the gas for constant heat, and the density of the gas; the velocity may also be calculated from the formula $v=\sqrt{\frac{p\gamma}{d}}$, where p is the pressure of the gas, d its density, and γ the ratio of its specific heat at constant pressure and constant volume. Reconcile these statements. N. S. Tripos. 1885.

26. Explain the reflection of sound at the end of an open and a closed organ-pipe, and deduce the possible notes for a pipe of given length.

Calculate the vibration frequency of a note sounded upon a closed organ-pipe 120 centimetres long, blown with air at a temperature of 15°C., knowing that the specific gravity of air at 0°C. and 760 mm. pressure is 0·001292, the specific gravity of mercury 13·59, the acceleration of gravity $981\frac{\text{cm.}}{(\text{sec.})^2}$, the coefficient of expansion of air ·00366, and the ratio of the two specific heats of air 1·406. N. S. Tripos. 1885.

27. What should be the length of a glass tube, open at both ends, that it may produce the maximum resonance when a tuning-fork, making 480 complete vibrations per second, is sounding near one end? State clearly the nature and relative extent of the motion of the air-particles in different parts of the tube, and show how the sound-waves are propagated in it. (Velocity of sound in air is 1120 feet per second.) Prel. Sc. 1887.

28. State the laws for the frequency of vibration of stretched strings.

A string of india-rubber is stretched by a force of 2 lbs., and it executes n transverse vibrations per second. The length is doubled when the stretching force is $3\frac{1}{2}$ lbs. What is now the frequency of the vibrations?

Vict. Int. 1886.

29. One end of a string is attached to a prong of a tuning-fork; the string passes over a small pulley, and carries a weight at the other end. With a weight of 40 grammes the string divides into four segments when the fork vibrates: what weight must be suspended to cause the string to vibrate in five and in six segments?

If the string be originally in the plane of vibration of the fork, what effect will be produced by turning the fork so that the plane of vibration of the prongs is at right angles to the string? B. Sc. 1885.

CHAPTER VIII

MAGNETISM

Note.—All quantities are expressed in terms of the C.G.S. units. For the definitions of magnetic units and their dimensions, see pp. 4 and 15.

1. A magnetic pole of strength 90 is found to attract another pole 2 cm. from it with a force equal to the weight of a gramme: what is the strength of the second pole?

By the law of inverse squares, the force exerted between the two poles is equal to the product of their strengths divided by the square of their distance apart. This is equal to 981 dynes (the weight of a gramme), so that if P be the strength of the second pole,

$$981 = P \times 90/2^2,$$

and $$P = 4 \times 981/90 = 43{\cdot}6.$$

2. The strength of a certain magnet-pole is 27: find the intensity of the magnetic field 3 cm. away from it, assuming the magnet to be so long that the influence of the other pole may be neglected. What force would be exerted by it upon a pole of strength 32 at a distance of 12 cm.?

The intensity of the magnetic field at any point is measured by the force experienced by a unit magnetic pole placed at the point. The force exerted by the given magnet-pole on an unit pole 3 cm. away from it is $27 \times 1/3^2 = 3$ dynes; and hence the strength of the field at this distance is 3.

In the second case the force would be $27 \times 32/(12^2) = 6$ dynes.

8. What is the force exerted between two poles of strength 32 and 36 units at a distance of 12 cm. from one another?

4. The repulsive force between two poles is 20 dynes when they are 4 cm. apart: what will it be when the distance between them is increased by 1 cm.?

5. A magnet-pole of strength 10 attracts another pole 5 cm. from it with a force of 2 dynes: what is the strength of the second pole?

6. The distance between two equal magnet-poles is 8 cm., and they repel one another with a force of 5 dynes: find the strength of each.

7. A magnet 8 cm. in length lies in a field of intensity $H = 0{\cdot}18$, and the strength of each of its poles is 5. Find the moment of the couple required to deflect it (1) through an angle of 30° from the magnetic meridian, (2) at right angles to the magnetic meridian.

The force acting on each pole in both cases is $mH = 5 \times 0{\cdot}18 = 0{\cdot}9$.

(1) The arm of the couple, or perpendicular distance between the forces acting on the two poles, is $l \sin \delta = 8 \sin 30^\circ = 8/2 = 4$, and the moment of the couple is $mHl \sin \delta = 0{\cdot}9 \times 4 = 3{\cdot}6$.

(2) When the needle is at right angles to the meridian the arm of the couple is equal to the length of the needle, and the moment of the couple is $0{\cdot}9 \times 8 = 7{\cdot}2$.

8. Given that the dimensions of strength of pole are $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$, show that the dimensions of strength of field are $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$. What is the strength of a pole which is urged with a force of 9 dynes when placed in a field of intensity 0·5?

9. A freely suspended magnetic needle is deflected (1) through an angle of 45°, (2) through an angle of 60° from the magnetic meridian. Compare the couples which tend to bring the needle back to its position of rest in the two cases.

10. Prove that the magnetic force (or intensity of field) due to a bar magnet at any point on the axis of the magnet produced is equal to $2Md/(a^2 - l^2)^2$, where M is the magnetic moment of the magnet, $2l$ its length, and d the distance of the point from its centre.

> Find the force due to each pole separately and subtract. Observe that since the length of the bar magnet is $2l$, its magnetic moment is $M = 2ml$.

11. Prove that the magnetic force due to the same magnet at a point opposite to its centre and at a distance d from it is equal to $M/(d^2 + l^2)^{\frac{3}{2}}$, and acts parallel to the axis of the magnet.

> The forces due to the two poles are equal, and their components perpendicular to the axis are equal and opposite. Find the components parallel to the axis and add.

12. A magnetic needle is deflected through an angle α from the meridian by a bar magnet of magnetic moment M placed "broadside on" (*i.e.* in the position shown in Fig. 3, the line joining the centres of the two magnets being in the meridian MR, and the axis of the bar-magnet perpendicular to it.) Prove that if H be the strength of the field, and d the distance between the centres of the magnets, $M/H = (d^2 + l^2)^{\frac{3}{2}} \tan \alpha$.

> Find the moments of the couples due to the action of the bar-magnet and of the earth's field, and equate.

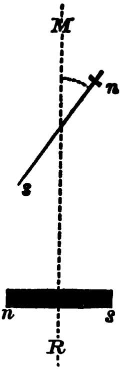


Fig. 3.

13. Prove that the equation of equilibrium in the preceding example would take the form

$$M/H = (d^2 - l^2)^2 \tan \alpha/2d,$$

if the bar-magnet were placed "end on," *i.e.* in an east and west position, with the centre of the deflected magnet lying on the axis of the bar-magnet produced.

Note on the Time of Vibration of a Magnet.—The small oscillations of a magnet are governed by laws similar to those which regulate the vibrations of a pendulum swinging through a small arc. The time of a complete vibration of a simple pendulum is $t = 2\pi\sqrt{l/g}$: the time of a complete vibration of a magnetic needle suspended horizontally is $t = 2\pi\sqrt{I/K}$, where I is the moment of inertia of the needle, and K is the moment of the directive force tending to restore the needle to its position of rest. The quantity I depends upon the mass and dimensions of the needle; K is the product of two factors, M and H, M being the magnetic moment of the magnet, and H the horizontal intensity of the earth's magnetic force. The force acting on a needle is therefore proportional to the square of the number of oscillations which it makes in a given time.

14. A magnetic needle is suspended horizontally at a considerable height above a bar-magnet which lies on the floor underneath it. When the north pole of the bar-magnet points southward, the needle makes 14 vibrations per minute; and when its north pole points northwards, the needle makes 8 vibrations per minute. At what rate would the needle vibrate under the action of the earth's force above?

Let n denote the number of oscillations made per minute by the needle under the action of the earth's force H. Then $H = kn^2$, where k is a constant.

In the first position of the bar-magnet the needle is vibrating in a field the strength of which is $H + F$, where F is the force due to the bar-magnet. In the second position of the bar-magnet its action is opposed to that of the earth, and the resultant field is of strength $H - F$.

Thus $$H + F = k \times (14)^2 = 196k,$$
and $$H - F = k \times (8)^2 = 64k.$$
$$\therefore\ H = 130k.$$

Again, since $H = kn^2$, it follows that $n = \sqrt{130} = 11{\cdot}4$.

15. A compass needle makes 50 oscillations per minute at a place where the dip is 64°, and 48 oscillations per minute at another place where the dip is 71°. Compare the value of the total magnetic force at the two places.

Let I denote the total force which acts along the line of dip, H the horizontal component, and θ the angle of dip; then $H/I = \cos\theta$, or $I = H/\cos\theta$ (Fig. 4). Thus at the first place we have $I = H/\cos 64^\circ \doteq H/0{\cdot}438$, and at the second place $I' = H'/\cos 71^\circ = H'/0{\cdot}326$.

The vibrations of the compass-needle are controlled by the *horizontal component* of the total force, therefore

$$H : H' = (50)^2 : (48)^2 = 2500 : 2304,$$

and

$$I : I' = (2500/0{\cdot}438) : (2304/0{\cdot}326)$$
$$= 1 : 1{\cdot}238.$$

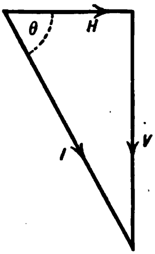


Fig. 4.

16. A magnetic needle makes 100 oscillations in 8 min. 20 sec. under the action of the earth's force alone. Under the combined action of the earth and a magnet A it makes 100 oscillations in 7 min. 30 sec., and when the magnet A is replaced by another B the needle makes 100 oscillations in 6 min. 40 sec. Compare the magnetic moments of A and B.

17. A compass needle makes 10 oscillations per minute under the influence of the earth's magnetism alone. When the north pole of a long magnet A is held 1 ft. south of it, the needle makes 12 oscillations per minute; and when the north pole of another magnet B is held in the same position, the number of oscillations per minute increases to 15. Compare the pole-strengths of the magnets A and B.

18. At Berlin the total magnetic intensity is 0·48 (in C.G.S. units) and the dip is 64°: at New York the total intensity is 0·61 and the dip 72°. If a magnet vibrating horizontally at Berlin makes 20 oscillations in a minute, how many oscillations would it make in the same time at New York?

19. A dip-circle is rotated (in azimuth) through an angle a from the magnetic meridian, and the apparent angle of dip under these conditions is θ': prove that the true dip (θ) at the place is given by the equation

$$\tan\theta = \tan\theta' \cos a.$$

20. Discuss the precise advantages of the method usually adopted for determining the magnetic dip (*i.e.* by observing the position in which the needle points vertically downwards, and then rotating the dip-circle through 90°); and prove that the true dip may be found from observations in *any* two azimuths at right angles by the formula

$$\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2,$$

θ_1 and θ_2 being the observed angles of dip in any two planes at right angles, and θ being the true dip.

EXAMINATION QUESTIONS.

21. Explain what is meant by the strength of a magnetic pole, and describe experiments to determine the law of force between two poles.

A circle is described round a small magnet in a plane which contains the axis of the magnet, with its centre coincident with that of the magnet. Discuss the changes in the intensity and direction of the force on a magnetic pole which is carried round the circle; and show how from observations at the points in which the axis of the magnet cuts the circle, and at the points on the circle midway between these, the law of force may be determined. Camb. Schol. 1885.

22. The centre of gravity of a dip needle does not quite coincide with its axis of suspension. Describe the operations necessary in order to eliminate the error which would otherwise arise in the measurement of the magnetic dip. Int. Sc. 1883.

23. The intensities of the earth's horizontal magnetic force at two different places can be compared by observing at each the deflection by the same magnet of a small compass-needle placed in the same position relatively to the magnet. Explain the method, and show how the result of the comparison would be effected by a diminution of the magnetic moment of the compass-needle or of the magnet respectively, occurring between the observations at the first station and those at the second.

Int. Sc. 1885.

24. Describe the principle of measurement employed in the Torsion-balance.

A magnet suspended by a fine vertical wire hangs in the magnetic meridian when the wire is untwisted. If on turning the upper end of the wire half round the magnet is deflected through 30° from the meridian, show how much the upper end of the wire must be turned in order to deflect the magnet 45° and 60° respectively.

Int. Sc. 1884.

25. A bar-magnet is suspended in a Torsion-balance by a wire without torsion. When the torsion head is turned through 360°, the bar is deflected 30° from the meridian. Through how many degrees must the torsion head be turned that the magnet may be in equilibrium at right angles to the meridian? Prel. Sc. 1887.

26. A magnetic needle makes a complete vibration in a horizontal plane in 2·5 seconds under the influence of the earth's magnetism only, and when the pole of a long bar-magnet is placed in the magnetic meridian in which the needle lies, and 20 cm. from its centre, a complete vibration is made in 1·5 seconds. Assuming $H = \cdot 18$ (C.G.S.), and neglecting the torsion of the fibre by which the needle is suspended, determine the strength of the pole of the long magnet. Int. Sc. Honours 1886.

27. Two small magnetic needles are placed with their centres at a distance r (great compared with their own lengths) from each other, so that the centre of one is in the prolongation of the axis of the other. Show that the couple exerted by one on the other is approximately equal to $\frac{2MM_1}{r^3}\left(1 + \frac{A}{r^2}\right)$, where M and M_1 are the magnetic moments of the needles, and A is a constant.

B. Sc. 1879.

28. Describe the magnetic behaviour of a piece of soft iron in a magnetic field of gradually increasing intensity, and give experimental methods by which the truth of your statements can be verified.

B. Sc. Honours 1884.

CHAPTER IX

ELECTROSTATICS

Note.—All quantities are expressed in terms of the C.G.S. units. For the definitions of the electrostatic units and their dimensions, see pp. 4 and 16.

1. Two small spheres are at a distance of 5 cm. apart: one has a charge of 10 units of electricity, the other a charge of 5 units. What is the force exerted between them?

It follows from Coulomb's law, and from the definition of the unit quantity of electricity, that the force (in dynes) is equal to the product of the charges divided by the square of the distance between the spheres.

Thus $F = 10 \times 5/5^2 = 50/25 = 2$ dynes.

If the two charges are of the same kind (*i.e.* both positive or both negative) the force will be one of repulsion; if the one charge is positive and the other negative, the force will be one of attraction.

2. Two small electrified bodies at a distance of 12 cm. apart are found to attract one another with a force of 6 dynes. The one has a positive charge of 32 units: what is the charge of the other?

3. What is the distance between two small spheres which have charges of 32 and 36 units respectively, and repel one another with a force of 8 dynes?

4. Express in dynes the repulsive force exerted between two small spheres 15 cm. apart, and charged respectively with 40 and 45 units electricity.

5. Two small spheres are 10 cm. apart, and one of them has a charge of 45 units: what must be the charge on the other so that the force exerted between them may be equal to the weight of 5 milligrammes?

6. Determine the relation between the electrostatic unit of quantity in the metre-milligramme-minute system and the corresponding C.G.S. unit.

7. An electrified ball is placed in contact with an equal and similar ball which is unelectrified: on being separated 8 cm. from one another the force of repulsion between them is equal to 16 dynes. What was the original charge on the electrified ball?

> Since the balls are of equal size the charge will be equally shared between them when they are placed in contact. Let q be the charge on each: then the repulsive force between them is $(q^2/8^2)$, and this is equal to 16 dynes. Thus $q^2 = 8^2 \times 16$, and $q = 8 \times 4 = 32$. The original charge on the electrified ball was $2q = 2 \times 32 = 64$ units.

8. Two small equal balls, one having a positive charge of 10 units and the other a negative charge of 5 units, are 5 cm. apart: what is the attractive force between them? If they are made to touch, and again separated by the same distance, what will be the force of repulsion?

9. Two small spheres, each charged with 50 units of electricity, are placed at two of the corners of an equilateral triangle 1 metre on the side: what is the magnitude and direction of the resultant electric force at the third corner?

10. What charge is required to electrify a sphere of 25 cm. radius until the surface-density of the electrification is $5/\pi$?

> The *surface-density* is the quantity of electricity per unit of surface. Thus if S be the area of the surface, and σ the surface-density, the charge is $Q = S\sigma$. The area of the

surface of a sphere of radius r is $4\pi r^2$: thus $S=4\pi \times (25)^2$, and $Q=4\pi \times (25)^2 \times 5/\pi = 20 \times 625 = 12500$.

11. A sphere of 5 cm. radius has a charge of 1000 units of electricity: what is the surface-density of the charge?

12. What charge must be imparted to a spherical conductor of 3 cm. diameter in order that the superficial density of the electrification may be 7? [Take $\pi = 22/7$.]

13. A magnetised knitting-needle, carrying a small gilt pith-ball at one end, is suspended horizontally by a silk fibre: a second pith-ball, of the same size as the first, is electrified and brought into contact with it. Prove that the charge on the second pith-ball is proportional to $\left(\sin \frac{\alpha}{2}\right)^{\frac{3}{2}}$, where α is the angle through which the knitting-needle is deflected from the magnetic meridian.

14. Three small electrified spheres, A, B, and C, have charges 1, 2, and 4 respectively. Find the position in which B must be placed between A and C in order that it may be in equilibrium. Prove also that there is another position along the line CA produced in which B will be equally repelled by A and C.

15. The bob of a seconds pendulum consists of a sphere of mass 16 grammes, and it is suspended by a silk thread. Vertically beneath it is placed a second sphere, which is positively electrified, and when the pendulum-bob is negatively electrified its time of oscillation is found to be 0·8 sec. Prove that the attractive force between the two spheres is equal to the weight of 9 grammes. (The arc of vibration is supposed to be so small that the attractive force is always along the vertical.)

Potential and Capacity.—It can be shown[1] that if a quantity q of electricity be collected at a given point, the difference of potentials due to it at any two given points A and B, whose distances from the given point are r and r' respectively, is

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[1] Clerk Maxwell, *Elementary Treatise on Electricity*, Art. 86; Silvanus Thompson, *Electricity and Magnetism*, Art. 238.

$$V_A - V_B = q/r - q/r'.$$

If the point B is removed to an infinite distance, or is connected with the earth, r' becomes ∞, and $q/r' = 0$. If we agree to regard the potential of the earth as zero, the expression for the difference of potentials between A and the earth, or, more briefly, the potential of the point A, reduces to

$$V_A = q/r.$$

If instead of a single quantity of electricity there are several charges q_1, q_2, q_3 . . . whose distances from the given point are r_1, r_2, r_3 . . . respectively, then the potential at A due to all these charges is

$$V_A = \frac{q}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ . \ . \ . \ = \Sigma \left(\frac{q}{r}\right).$$

The external action of an electrified spherical conductor is the same as if all the charge were collected at its centre. If the charge be Q, the potential due to it at any external point, whose distance from the centre of the sphere is r, is Q/r. This is only true when r is not less than the radius R of the sphere. At the surface of the sphere $r = R$, and the potential is Q/R. Now the capacity (C) of the sphere is measured by the charge required to raise its potential from zero to unity, or

$$C = Q/V,$$

where Q is the charge and V the potential due to it. But we have seen that $V = Q/R$. Hence $C = R$, or—

The capacity of a spherical conductor (placed in air at a considerable distance from other conductors) is numerically equal to its radius.

16. A hollow spherical conductor, whose radius is 1 decimetre, is charged with 10 units of electricity: find the potential (1) at the surface of the sphere, (2) inside it, (3) at points distant 15 and 25 cm. from its centre. If the sphere is connected by a long thin wire with a second conducting sphere of 1 cm. radius, what will be the charge and potential of each sphere?

(1) At the surface $V = Q/C = Q/R = 10/10 = 1$.

(2) The potential is constant and equal to unity throughout the whole interior of the sphere.

(3) At the first point $V = 10/15 = 0\cdot\dot{6}$; at the second $V = 10/25 = 0\cdot4$.

After connection is made the two spheres will be reduced to a common potential. The capacity of the system (neglecting the capacity of the fine wire) is equal to the sum of the capacities of the two spheres $= 10 + 1 = 11$. Their joint charge is equal to the original charge $= 10$. Hence their common potential is $V = Q/(C + C') = 10/11 = 0\cdot9\dot{0}\dot{9}$. The charge of the first sphere is $q_1 = CV = 10 \times 10/11 = 9\cdot\dot{0}\dot{9}$; the charge of the second sphere is $q_2 = C'V = 1 \times 10/11 = 0\cdot9\dot{0}\dot{9}$.

Observe that the charge is shared in the ratio of 10 to 1; *i.e.* in the ratio of the capacities of the conductors.

17. Charges of 50 units of electricity are placed at each of the corners of a square whose side is 1 metre: find the potential at the point of intersection of the diagonals.

18. Two conductors, of capacity 10 and 15 respectively, are connected by a fine wire, and a charge of 1000 units is divided between them: find the charge which each takes, and the potential to which it is raised.

19. A conductor of capacity 75 is charged to a potential 20, and is then made to share its charge with a second conductor of capacity 25: what will be the final charge and potential of each?

20. Three spheres of capacity 1, 2 and 3 are charged to potentials 3, 2 and 1 respectively, and are then connected by a fine wire: what is their common potential?

21. Find the dimensions of capacity in the electrostatic system, and calculate the capacity of a sphere of 5 metres radius, in that system in which the decimetre, gramme, and minute are taken as units.

22. Two small uncharged metallic spheres are hung up by silk threads in an electrical field; they are connected together for a moment by a fine wire, which is then removed. If the spheres are now examined, what will be their electrical state? If no charge is found upon either of the spheres, what conclusion would you draw as to the relative potentials of the points occupied by the centres of the spheres at the instant when the wire was removed?

23. Two spheres, each of 1 cm. diameter, are connected by a wire, and are at the same potential 40. The force of repulsion between them is 5 dynes: what is their distance apart? What will be the force when the distance between them is half a metre?

24. What is an equipotential surface? Show that the work done by or against the electrical forces during the [illegible] of a charged body from one equipotential surface to another is independent of the path along which the body moves.

25. Two spheres, of [illegible] and 3 cm. radius, are charged [illegible] and 10: what will be their [illegible] they are placed in electrical con-

26. [illegible] conductor is charged with [illegible] over its surface? [illegible] the lines of force and [illegible] neighbourhood. Explain [illegible] the charge will be [illegible] sphere an uncharged [illegible] insulated, (2) when it is

27. Two spheres, of 2 and 6 cm. radius, are charged respectively with 80 and 30 units of electricity: compare their potentials. If they are connected by a fine wire, how much electricity will pass along it?

28. Two small electrified spheres of the same size are placed at a distance of 4 cm. apart, and the charge of the one is double that of the other. The two spheres are brought into contact, and are then removed to a distance of 6 cm. from one another: find the ratio between the repulsive forces in the two cases.

29. A charged sphere of radius r is made to share its charge with a second sphere of radius r'; prove that the density of the electrification on the first is to that on the second as $r' : r$. (See Ex. 10.)

30. The diameters of two spheres are 4 cm. and 6 cm. respectively, and the potential of the second is one-third greater than that of the first: compare the surface-densities of their charges.

31. A small sphere, charged with 4 units of positive electricity, is 12 cm. from a second insulated sphere charged with 9 units of negative electricity. The second sphere is removed, touched with an unelectrified sphere of one half its diameter, and is then placed 8 cm. from the first sphere. Prove that the attractive forces in the two cases are as 2 : 3.

32. A sphere of 25 cm. radius is charged until its surface-density is $5/\pi$: what is its potential?

33. A number of small insulated spheres situated on the circumference of a circle are charged to different potentials: prove that the potential at the centre of the circle will not be altered if all the spheres are connected together by fine wires whose capacity may be neglected.

34. Two small insulated bodies, of capacities a and b, receive charges A and B respectively. What will be their common potential if they are connected by a long fine wire, and how much electricity will flow along the wire?

35. In the preceding question, find the point on the line joining the two small bodies at which the potential is the same before and after contact is made by means of the wire.

Capacity of a Spherical Air-Condenser.—The capacity of a spherical air-condenser, consisting of an insulated sphere of radius r, and a concentric spherical shell of radius r', is $rr'/(r'-r)$.

For let M (Fig. 5) be the common centre, and suppose a charge Q of positive electricity ("+ Q") to be imparted to the inner sphere A. This will induce an equal negative charge, – Q, upon the internal surface of the outer spherical shell B. If B is insulated there will also be a charge + Q repelled to its outer surface, but this positive induced charge can be removed by placing B in contact with the earth. The potential of the outer sphere will now be zero, and so also will be the potential at any external point N.

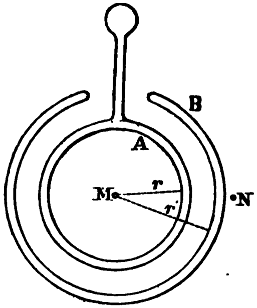

Fig. 5.

For if d be the distance of N from M, the potential at N due to the charge on A (see p. 192) is $+Q/d$: the potential at N due to the charge on B is $-Q/d$, and the sum of these two quantities is zero.

At the surface of A the potential due to its own charge is Q/r, and that due to the charge on B is $-Q/r'$ (for the potential due to the charge on the outer sphere B at any point inside it is constant, and is equal to the potential at the surface of B due to this charge).

Thus the resultant potential of A is $Q/r - Q/r'$. The capacity of the condenser is numerically equal to the charge required to raise the potential of the inner sphere from zero to unity, or

$$C = \frac{Q}{Q/r - Q/r'} = \frac{1}{1/r - 1/r'} = \frac{rr'}{r' - r}.$$

Specific Inductive Capacity.—The *Specific Inductive Capacity* of a substance is the ratio between the capacity of a condenser containing the given substance as dielectric and that of an equal and similar air-condenser. If the capacity of the latter be C, then the capacity of the former is $C' = Ck$, when k is the specific inductive capacity of the dielectric which it contains. If the condenser consist of two concentric spherical surfaces, the full expression for its capacity is $C' = k.rr'/(r' - r)$.

36. An uncharged condenser containing a solid dielectric is placed in electrical connection with an equal and similar air-condenser charged to the potential V. If the common potential after sharing the charge is V′, what is the specific inductive capacity of the dielectric?

If C be the capacity of the air-condenser, its original charge was $Q = VC$. The joint capacity of the system after connection is made is $(C + C')$, where C' is the capacity of the uncharged condenser; and the total charge is $V'(C + C')$. This is also equal to Q, for no electricity is supposed to be lost in sharing the charge.

$$\therefore \; VC = V'(C + C').$$

If k be the specific inductive capacity of the dielectric, $C' = Ck$,

and $$VC = V'C(1 + k),$$

or $$k = (V - V')/V'.$$

37. A charge Q is imparted to an uninsulated air-condenser consisting of two concentric spherical surfaces of radii r and r'. Prove that at any point between the two surfaces, and at a distance d from their common centre, the value of the potential is $Q/d - Q/r'$.

38. Find the capacity of a spherical condenser, the inner coating of which has a diameter of 60 cm., and the dielectric being glass of specific inductive capacity 6·8 and thickness 1·5 cm.

39. Two condensers are similar in every respect, excepting that one contains air as the dielectric, and the other turpentine of specific inductive capacity 2·16. If a charge of 500 units is divided between them, what charge will each take?

40. Two metal spheres, whose radii are 10 cm. and 5 cm. respectively, are connected by a long fine wire, and the smaller sphere is surrounded by a concentric spherical shell of 5·5 cm. internal radius. If a charge of 520 units is divided between them, what will be the charge and potential of each?

41. A condenser consists of two concentric spherical surfaces whose radii are 100 mm. and 101 mm. respectively. The space between them is filled with sulphur of specific inductive capacity 3·4. Find the radius of an insulated spherical conductor which would have the same capacity as this condenser.

Energy of a Charged Condenser.—It follows from the definition of difference of potential that if Q units of electricity are transferred from a place at which the potential is V to another place at which the potential is V′, the work done by the electrical force is $Q(V - V')$ ergs: if $V' = 0$ (as, for example, if the charge is removed to an infinite distance, or allowed to escape to earth), then the work done is QV. But all this is on the supposition that the potential at both places remains constant and is unaltered by the transference of the charge; which would be approximately correct if the charge were transferred from one conductor of large dimensions (and therefore large capacity) to another conductor of large dimensions, such as the earth.

The case is different when the potential of the charged body is due to its own charge; for if the conductor or condenser is of finite capacity C, and if a charge Q is imparted to it, it is raised to a definite potential V, which is given by the equation $C = Q/V$. If the electricity is allowed to escape gradually to earth, then as the charge diminishes from Q to zero, so the potential falls from V to zero, the *average* potential during the process being V/2. It can be proved that the work done in the discharge is measured, not by the product of the charge into the potential, but by the product of the charge into the *average* potential during the discharge, and is therefore equal to QV/2.

For the discharge may be supposed to take place by n separate steps, such that in each of these separate discharges a quantity q of electricity is removed. The potential is Q/C at the beginning, and $(Q-q)/C$ at the end of the first discharge; hence the work done in this first discharge is *less than* qQ/C and *greater than* $q(Q-q)/C$. The difference between these quantities is q^2/C, and the difference between either of them and the correct expression for the work done in this partial discharge is *less than* q^2/C.

The potentials at the beginning of the first, second, nth discharges are Q/C, $(Q-q)/C$, . . . $(Q-\overline{n-1}\cdot q)/C$ respectively. The amounts of work done in these separate discharges may be represented by qQ/C, $q(Q-q)/C$, . . . $q(Q-\overline{n-1}\cdot q)/C$ respectively. If we add together the n terms so as to find the total work done in the discharge, we shall make an error which is *less than* $n \times q^2/C$. Now the whole charge Q is equal to $n \times q$, so that the error is less than qQ/C; and if n be made very large q becomes so small that the quantity qQ/C may be neglected.

The total work done is

$$W=\frac{q}{C}\left\{Q+(Q-q)+\ .\ .\ .\ (Q-\overline{n-1}\cdot q)\right\},$$

$$=\frac{q}{C}\left\{nQ-(1+2+\ .\ .\ .\ +\overline{n-1})q\right\}.$$

Now the sum of the first n natural numbers is $n(n+1)/2$, and the sum of the first $(n-1)$ natural numbers is $(n-1)n/2$.

Thus
$$W=\frac{Q^2}{C}-\frac{(n-1)nq^2}{2C}=\frac{Q^2}{2C}+\frac{Qq}{2C},$$

and, as we have seen, the last term can be neglected when n is made very large.

Since $Q=VC$, the energy of the charge (*i.e.* the work done in the discharge) is equal to $Q^2/2C$, or $QV/2$, or $CV^2/2$.

The work done against the electrical forces in charging the conductor or condenser is also equal to $\frac{1}{2}QV$. This follows necessarily from the principle of the conservation of energy: it may also be proved independently by a method similar to that which we have adopted above.

42. A Leyden jar of capacity C_1 is charged and discharged. It is again charged to the same potential, and partially discharged by allowing it to share its charge with an empty jar of capacity C_2. Lastly, the two jars are separately discharged. Compare the energies of the four discharges.

The energy of the original charged jar is $Q^2/2C_1$, Q denoting the quantity of electricity which it contains.

The same quantity is next shared between the two jars whose joint capacity is C_1+C_2, so that their common potential after the charge is shared is $V=Q/(C_1+C_2)$.

The energy of the charge of the first jar is now

$$\frac{C_1V^2}{2}=\frac{C_1}{2}\left(\frac{Q}{C_1+C_2}\right)^2,$$

and that of the second jar is

$$\frac{C_2V^2}{2}=\frac{C_2}{2}\left(\frac{Q}{C_1+C_2}\right)^2.$$

The energy lost in the partial discharge (*i.e.* in sharing the charge) is

$$\frac{Q^2}{2C_1}-\frac{C_1+C_2}{2}\cdot\left(\frac{Q}{C_1+C_2}\right)^2=\frac{C_2Q^2}{2C_1(C_1+C_2)}.$$

Thus the energies of the four discharges, taken in the order in which they occur, are as

$$\frac{1}{C_1}:\frac{C_2}{C_1(C_1+C_2)}:\frac{C_1}{(C_1+C_2)^2}:\frac{C_2}{(C_1+C_2)^2},$$

i.e. as $(C_1+C_2)^2 : C_2(C_1+C_2) : C_1^2 : C_1C_2$.

43. An air-condenser, whose armatures are concentric spheres of diameter 20 and 24 cm. respectively, is charged to potential 50: find the work done in charging it.

44. A condenser of capacity 10 is raised to a potential 30: what is its charge, and how much work is done in charging it?

45. A spherical air-condenser, the coatings of which have radii of 16 and 17·5 cm. respectively, is charged to a potential 12, its outer coating being in contact with earth. Calculate its energy.

46. Two insulated spheres, of 12 cm. and 3 cm. radius, are charged respectively with 36 and 24 units of electricity: compare their potentials, and the energies of their charges.

47. Compare the work done in charging a Leyden jar of capacity 30 to potential 15 with that required to charge a jar of capacity 20 to potential 45.

48. The armatures of a condenser are concentric spheres of diameter 20 and 24 cm. respectively, and the space between them is filled with shellac of S.I.C. = 3. The condenser has a charge of 7560 units. If it is discharged in such a way that all the energy is converted into heat, how much heat will be produced? [J = 4·2 × 10^7.]

The capacity of the condenser is $3 \times 10 \times 12/2 = 180$. The energy of its charge is $Q^2/2C = (7560)^2/360 = 158760$ ergs.

Let H denote the amount of heat (in terms of the calorie or gramme-degree) resulting from the discharge: then JH = E, or

$$4{\cdot}2 \times 10^7 \times H = 158760,$$

and $\therefore H = 0{\cdot}00378$ calorie.

49. The capacity of a condenser is 700; to what potential must it be charged in order that the energy of its discharge may be equivalent to one heat-unit?

50. Two equal and similar Leyden jars have their outer coatings connected to earth: one is uncharged, the other is charged to potential V. Find the energy of each after the charge has been shared, and show that one-half of the energy of the charged jar is dissipated in the spark which passes when their knobs are connected.

51. The coatings of a charged Leyden jar are connected with those of an uncharged jar of double the capacity. Compare the energy of the system with that of the original charged jar.

52. What would be the solution of the preceding example if the linear dimensions of the uncharged jar were double those of the charged jar, other things being equal?

53. A charged sphere of radius r is made to share its charge with an uncharged sphere of radius r': prove that the energy of the original distribution is to that of the final distribution as $(r+r') : r$.

54. A Leyden jar is charged and then connected up with nine uncharged jars so as to form a battery of ten equal jars: show that the energy of the whole battery is only one-tenth that of the single jar.

55. A condenser is charged to a given potential and then discharged. It is again charged to the same potential, and made to share its charge with another condenser of half its capacity, after which the jars are separately discharged. Compare the energy of discharge in each case.

56. A Leyden jar is charged with electricity; an equal charge is imparted to a battery consisting of four equal and similar jars, the inner coatings of which are connected together. Compare the energies of the two charges.

57. The outer armatures of two spherical air-condensers are of the same radius, viz. 20 cm. The first is charged, and the radius of its inner armature is 15 cm.: the second is uncharged, and its inner armature has a radius of 18 cm. Prove that if the first condenser is made to share its charge with the second, three-fourths of its energy will be lost.

58. A given quantity of electricity is to be shared between a number of conductors of different capacities. Prove that the energy of the system is a minimum when all the conductors are charged to the same potential.

EXAMINATION QUESTIONS.

59. Define the potential and the capacity of a charged conductor. Two insulated hollow conducting spheres of radii a and b $(a > b)$ are charged at a considerable distance from one another to potentials A and B. The larger is then opened and the other is put inside and allowed to touch. Determine the potentials of the spheres and the quantities of electricity in each.

Edinb. M.A. 1884.

60. Of two similar metal discs A and B, placed parallel to each other, A is connected with a gold-leaf electroscope, and B with the gas-pipes. A small charge of electricity is given to A, and the leaves of the electroscope diverge. When a slab of sulphur is introduced between the discs, the divergence diminishes. But if B be insulated and charged, while A is charged only by induction, the introduction of the sulphur causes an increase of the divergence. Explain these experiments.

Prel. Sc. 1887.

61. Two small equal spheres, A and B, placed with their centres at a distance of 1 metre apart, are charged with 25 and -25 units of electricity respectively. Find the direction and magnitude of the resultant electric force at a point 1 metre from each of the spheres. Find also the electric potential at the same point.

Prel. Sc. 1888.

62. Assuming that the quantity of electricity produced by a plate machine is proportional to the number of turns of the disc, explain how the capacities of two condensers may be compared. Int. Sc. 1883.

63. A, B, and C are three Leyden jars, equal in all respects. A is charged, made to share its charge with B, and afterwards share the remainder with C, both B and C being previously without charge. The three jars are now separately discharged. Compare the quantity of heat resulting from each discharge with what would have been produced by the discharge of A before any sharing of its charge. Int. Sc. 1884.

64. What must be the velocity of a bullet of 15 gm. that its kinetic energy may be equal to the electric energy of a globular flask of 8 cm. radius, half filled with oil of vitriol and half immersed in oil of vitriol, the glass being 0·05 cm. thick, its specific inductive capacity = 6, the liquid inside being at potential 300, and the liquid outside at potential 700? Int. Sc. Honours 1884.

65. Explain the action of the attracted disc electrometer. A circular plate connected with the earth, 5 cm. in radius, hangs from a balance at a distance of ·5 cm. above an equal horizontal disc which is insulated. On electrifying the lower disc, a mass of 8 gm. has to be placed in the other pan of the balance to maintain equilibrium. Find the potential of the lower disc. In what units is your answer expressed? Int. Sc. Honours 1887.

CHAPTER X

CURRENT ELECTRICITY

Ohm's Law.—The current which flows along any conductor is directly proportional to the electromotive force (or difference of potential) between its ends, and is inversely proportional to its resistance. Thus if C denote the current and E the electromotive force (or E.M.F.),

$$C \propto \frac{E}{R}, \text{ or } C = k \cdot \frac{E}{R},$$

R being the resistance of the conductor, and k a constant.

If we agree to define the resistance of a conductor as being the ratio between the E.M.F. along it and the current thereby produced ($R = E/C$), the constant k becomes equal to unity, and Ohm's law may be expressed by the equation

$$C = \frac{E}{R}.$$

This equation holds good when C, E, and R are expressed in terms of the C.G.S. electromagnetic units defined on p. 5, and also when these three quantities are expressed in terms of the so-called practical units (current in ampères, E.M.F. in volts, and resistance in ohms).

1. An incandescent lamp takes a current of 0·7 ampère, and the E.M.F. between its terminals is found to be 98 volts: what is its resistance?

Since C is expressed in ampères and E in volts, the resistance of the lamp, in ohms, will be given by the equation

$$R = E/C = 98/0{\cdot}7 = 140.$$

2. The E.M.F. of a battery (or difference of potential between its poles on open circuit) is 15 volts: when the poles are connected by a copper wire a current of 1·5 ampère is produced, and the potential difference between the battery poles falls to 9 volts. Find the resistance of the wire and the internal resistance of the battery.

The resistance (R) of the wire is the ratio of the difference of potential between its ends (9 volts) to the current thereby produced (1·5 ampère),

$$\textit{i.e.}\ R = V/C = 9/1{\cdot}5 = 6 \text{ ohms}.$$

Notice that 9 volts is the potential difference causing the flow of current through the wire of resistance to 6 ohms; the *total E.M.F.* acting round the circuit is 15 volts. Call this E, and the resistance of the battery B: applying Ohm's law to the complete circuit, we have $E = C(B + R)$,

$$\textit{i.e.}\ 15 = 1{\cdot}5(B + 6) = 1{\cdot}5B + 9,$$

and $$B = 6/1{\cdot}5 = 4 \text{ ohms}.$$

3. The wire used on Indian telegraph lines is iron wire of No. 2 B.W.G., having a resistance of 4·6 ohms per mile. The batteries consist of Minotto cells of 1·04 volt E.M.F. and 30 ohms resistance per cell. Assuming that the resistance of the instruments is 80 ohms, and that a current of 8 milli-ampères is required to work them, find how many cells should be employed on a line 200 miles in length.

If n be the number of cells required, the E.M.F. of the battery is $1{\cdot}04\,n$ volt, and its internal resistance is $30\,n$ ohms. The resistance of the line is $4{\cdot}6 \times 200 = 920$ ohms, and that of the instruments is 80 ohms; the total external resistance (assuming that of the return circuit through the earth to be negligible) is 1000 ohms.

A *milli-ampère* is one-thousandth of an ampère, so that the required current is 0·008 ampère. By Ohm's law,

$$C = 1{\cdot}04\,n/(30\,n + 1000), \text{ and this} = 0{\cdot}008,$$

$$\therefore\ 1{\cdot}04\,n = 0{\cdot}24\,n + 8,$$

$$\textit{i.e. } 0{\cdot}8\,n = 8, \text{ and } n = 10.$$

4. A current of 8·5 ampères flows through a conductor, the ends of which are found to have a difference of potential of 24 volts: what is its resistance?

5. If an incandescent lamp of 80 ohms resistance takes a current of 0·75 ampère, what E.M.F. is required to work it?

6. A glow lamp takes a current of 1·32 ampère and the E.M.F. between its terminals is found to be 66 volts: what is its resistance while hot?

7. A battery consists of 5 Daniell cells, each having an E.M.F. of 1·08 volt and an internal resistance of 4 ohms: what current will the battery produce with an external resistance of 7 ohms?

8. You are required to send a current of 2 ampères through an electromagnet of 3·5 ohms resistance, and are supplied with a number of Grove cells each of 1·9 volt E.M.F. and 0·25 ohm internal resistance: how many cells are required?

9. One end (A) of a wire ABC is connected to earth; the other end (C) is kept at a constant potential of 100 volts. If the resistance of the portion AB is 9·6 ohms and that of BC 2·4 ohms, what current will flow along the wire, and what will be the potential of the point B?

10. A Bunsen cell has an internal resistance of 0·3 ohm and its E.M.F. on open circuit is 1·8 volt. The circuit is completed by an external resistance of 1·2 ohm: find the current produced and the difference of potential which now exists between the terminals of the cell. [See Ex. 2.]

11. On adding 3 ohms to the resistance of a certain circuit the current is diminished in the ratio of 6 to 5: what was the original resistance, and how much should be added to this in order to bring the current down to half its original value?

12. How many cells, each of 1·8 volt E.M.F. and 1·1 ohm internal resistance will be required to send a current of 0·5 ampère through an external resistance of 50 ohms?

13. Four Grove cells, each having a resistance of 0·25 ohm, are connected up with an electromagnet of 5 ohms resistance, and the current has only half the required strength: how many additional cells of the same kind must be used to produce the desired effect?

14. Two cells, of E.M.F. 1·8 volt and 1·08 volt respectively, are placed in a certain circuit in opposition (*i.e.* with their poles in such positions that the cells tend to send currents in opposite directions). The current is found to be 0·4 ampère: what current will be produced if the cells are placed properly in series?

15. The poles of a battery of 5 cells are connected by a wire 8 metres long, having a resistance of 0·5 ohm per metre; each cell has an E.M.F. of 1·4 volt and a resistance of 2 ohms: find the distance between two points on the wire such that the E.M.F. between these points is 1 volt.

16. The poles of a battery of 4 cells are connected by a wire 8 ft. in length, and the resistance of each cell of the battery is equal to that of 1 foot of the wire. Compare the E.M.F. acting along a portion of the wire, 3 ft. in length, with the E.M.F. of a single cell on open circuit.

17. A current is sent by a battery of constant E.M.F. (1) through a resistance of 20 ohms, (2) through a wire of unknown resistance, and (3) through a resistance of 40 ohms. The currents produced are in the ratio of 10 : 9 : 8; find the resistance of the battery and that of the wire.

18. The poles of a battery consisting of 40 Daniell cells in series are connected by a resistance of 280 ohms, and the current produced is 0·0535 ampère; when the external resistance is increased to 1080 ohms the current is reduced to one half: find the average resistance and E.M.F. of each cell of the battery, and determine the difference of potential existing between the poles of the battery when the external resistance is 280 ohms.

19. A Bunsen cell of 1·8 volt E.M.F. and a Planté cell (or accumulator) are connected up with a resistance of 400 ohms, the cells being in opposition, and the strength of the current is observed. On rearranging the cells, so that they tend to send currents in the same direction, it is found that the resistance has to be increased to 4000 ohms in order to reduce the current to its former value. Assuming that the resistances of the cells may be neglected, find the E.M.F. of the accumulator and the current produced.

20. With an external resistance of 9 ohms a certain battery gives a current of 0·43 ampère: when the external resistance is increased to 32 ohms the current falls to 0·2 ampère. What is the resistance of the battery?

21. The external resistance in a certain circuit is two-thirds that of the battery: what change will be produced in the current by reducing the internal resistance to half its former value, the E.M.F. remaining unchanged?

22. A No. 7 Brush dynamo gives an E.M.F. of 830 volts when running at the rate of 750 revolutions per minute, and its internal resistance is 11 ohms: show that such a machine can supply 16 arc lamps in series, each lamp offering a resistance of 4·5 ohms and requiring a current of 10 ampères.

23. The instruments on a telegraph circuit have a resistance of 230 ohms, and the line itself offers a resistance of 13 ohms per mile: how many Daniell cells, each of E.M.F. = 1·06 volt and internal resistance = 4 ohms, will be required to send a current of 10 milli-ampères through 100 miles of such a line?

24. The current along a telegraph line is tested at two stations whose respective distances from the sending battery are 50 and 150 miles. The current in the latter case is one-half that in the former. If the galvanometer has a resistance equal to that of 15 miles of the line wire, prove that the battery resistance is equal to that of 35 miles of wire.

Resistance.—The resistance of a conductor (of uniform section) is directly proportional to its length and inversely proportional to the area of its cross-section.

If l_1 and l_2 are the lengths of two uniform conductors made of the same material, s_1 and s_2 the areas of their cross-sections, the ratios of their resistances (R_1 and R_2) is

$$\frac{R_1}{R_2}=\frac{l_1/s_1}{l_2/s_2}=\frac{l_1 s_2}{l_2 s_1} \quad . \quad . \quad . \quad . \quad (1)$$

If the conductors are cylindrical wires of radii r_1 and r_2 respectively, then $s_1=\pi r_1^2$, and $s_2=\pi r_2^2$,

$$\therefore \frac{R_1}{R_2}=\frac{l_1 r_2^2}{l_2 r_1^2} \quad . \quad . \quad . \quad . \quad (2)$$

The resistance of a conductor further depends upon the material of which it is made. The *specific resistance* of a substance in the C.G.S. system is defined to be the resistance between the opposite faces of a cube, 1 cm. in the side, made of the substance.

If ρ be the specific resistance of a conductor of length l and cross-section s, its resistance, *in C.G.S. units*, is $\rho \times l/s$: expressed in *ohms* its resistance is $\rho \times l/s \times 10^9$, for one ohm $= 10^9$ C.G.S. units of resistance (p. 6).

If the conductor is a cylindrical wire of radius r its resistance is

$$R=\rho l/\pi r^2 \quad . \quad . \quad . \quad . \quad (3)$$

TABLE OF (C.G.S.) SPECIFIC RESISTANCES.

Silver	1,520
Copper	1,600
Aluminium	2,900
Platinum	9,000
Iron	9,700
Lead	19,500
Mercury	94,340

The numbers in the above table are only given for the purposes of calculation, as representing approximately the specific resistances of pure metals at 0°: the resistance of a conductor depends to some extent upon its physical state, and commercial metals usually contain impurities which considerably increase their resistance.

The specific resistance of a metal can be determined by measuring the resistance of a length l of wire made of the metal and then finding its diameter: this can be done directly by a micrometer wire-gauge, or, more accurately, by weighing the wire in air and in water so as to find its mass m and density δ. The cross-section of the wire is then given by the equation $(\pi r^2)l\delta = m$, from which we have $\pi r^2 = m/l\delta$.

By equation (3) the resistance of the wire is $R = \rho . l/\pi r^2$,

$$\left.\begin{aligned} \therefore\ R &= \rho . l^2\delta/m, \\ \text{and} \quad \rho &= mR/l^2\delta \end{aligned}\right\} \qquad (4)$$

If R_1 and R_2 are the resistance of two wires made of the same substance, l_1 and l_2 their lengths, m_1 and m_2 their masses, then

$$\frac{R_1}{R_2} = \frac{l_1^2/m_1}{l_2^2/m_2} = \frac{l_1^2 m_2}{l_2^2 m_1} \qquad (5)$$

25. Find the resistance at 25° of a copper wire 10 metres long and 1 mm. in diameter. The resistance of copper increases by 0·39 per cent for each degree rise in temperature.

The resistance of the wire at 0° is

$$R_0 = 1600 \times 1000/\pi \times (0{\cdot}05)^2,$$
$$= 2{\cdot}037 \times 10^8 \text{ C.G.S. units} = 0{\cdot}2037 \text{ ohm.}$$

At 25° the resistance is

$$R_{25} = 0{\cdot}2037(1 + \overline{0{\cdot}0039 \times 25}) = 0{\cdot}2236 \text{ ohm.}$$

26. A uniform glass tube 92·1 cm. in length was filled with mercury, and the resistance of the column of mercury was measured and found to be 1·059 ohm; the weight of the mercury contained in the tube was 10·15 gm. Calculate from this experiment the specific resistance of mercury, taking its sp. gr. as 13·6.

Expressed in C.G.S. units the resistance of the column is $1{\cdot}059 \times 10^9$, and therefore by equation (4) the specific resistance is

$$\rho = 10{\cdot}15 \times 1{\cdot}059 \times 10^9/(92{\cdot}1)^2 \times 13{\cdot}6 = 93177.$$

27. Two wires of the same length and material are found to have resistances of 4 and 9 ohms respectively: if the diameter of the first is 1 mm., what is the diameter of the second?

28. The resistance of a bobbin of wire is measured and found to be 68 ohms: a portion of the wire 2 metres in length is now cut off, and its resistance is found to be 0·75 ohm. What was the total length of wire on the bobbin?

29. Compare the resistances of two wires A and B, given that they are of the same weight and material, but that B is nine times as long as A.

30. Copper wire one-twelfth of an inch in diameter has a resistance of 8 ohms per mile: what is the resistance of a mile of copper wire the diameter of which is $\frac{1}{36}$ in.?

31. Calculate the resistance of a lead wire 5 metres long and 1 mm. in diameter.

32. If copper wire of No. 21 B.W.G. (diameter $= 0{\cdot}032$ in.) has a resistance of 54 ohms per mile, what is the resistance of a mile of No. 13 copper wire (diameter $= 0{\cdot}096$ in.)?

33. A mile of telegraph wire 2 mm. in diameter offers a resistance of 13 ohms; what is the resistance of 440 yards of wire 0·8 mm. in diameter made of the same material?

34. The resistance at 0° of a column of mercury 1 metre in length and 1 sq. mm. in cross-section is called a "Siemen's Unit." Find the value of this unit in terms of the ohm.

35. Copper wire of No. 20 on the new standard wire-gauge has a resistance of 0·026 ohm per metre, and weighs 5·84 gm. per metre: what is the resistance of a metre of No. 32 S. W. G. copper wire weighing 0·524 gm.?

36. Compare the resistances of two wires, one of which weighs 20·5 gm. and is 4·5 metres long, while the other weighs 82 gm. and is 18 metres long.

37. A trough 2 cm. deep and 2·5 cm. broad is cut in a wooden board. The trough is 2 metres long and is half filled with mercury: find the resistance between its ends.

38. What length of platinum wire 1 mm. in diameter is required in order to make a 1-ohm resistance coil?

39. A wire m metres in length and $1/n$th of a millimetre in diameter is found to have a resistance r: what is the specific resistance of the material of which it is made?

40. Find the resistance of an iron wire ABC which consists of two parts, the first (AB) being 60 cm. long and 1 mm. in diameter, and the second (BC) being 3 metres long and 1·6 mm. in diameter.

41. The poles of a battery are connected by a wire whose resistance is equal to that of the battery, and the poles of a second exactly similar battery are connected by a wire of the same weight and material, but three times as long as the first. Compare the currents in the two cases.

42. Find the length of an iron wire one-twentieth of an inch in diameter which will have the same resistance as a copper wire one-sixtieth of an inch in diameter and 720 yards long, the conducting power of copper being six times that of iron.

43. Express in microhms (or millionths of an ohm) the resistance of a strip of silver 10 cm. long, 0·5 cm. broad, and 0·1 mm. thick.

44. A wire made of platinoid (German-silver containing a small percentage of tungsten) is found to have a resistance of 0·203 ohm per metre. The cross-section of the wire is 0·016 sq. cm.: express the specific resistance of platinoid in microhms.

45. The density of aluminium is 2·7: what is the resistance of an aluminium wire 1 metre long and weighing 1 gm.?

46. The central conductor of the Bessbrook and Newry electrical tramway is made of steel having a specific resistance of 0·0000121 ohm, and costing £7 : 10s. per ton : high conductivity copper of 0·0000016 ohm specific resistance would have cost £84 per ton. The cross-section of the steel conductor was 8·817 sq. cm.: calculate the cross-section of a copper conductor of the same resistance, and show that it would have cost half as much again as the steel conductor.

The Tangent Galvanometer.—A tangent galvanometer consists of a circular coil of wire placed with its plane in the magnetic meridian, and at the centre of which is suspended a small magnetic needle.

The force exerted by a current flowing through the coil upon a magnet pole placed at (or very near to) its centre is directly proportional to the current-strength, the total length of the coil and the strength of the pole, and is inversely proportional to the square of the distance between each element of the circuit and the pole.

Let r be the radius of the coil, n the number of turns, and m the pole-strength of the magnet: then if the current C is expressed in terms of the C.G.S. electro-magnetic unit (as defined on p. 5), the force exerted upon *each* pole of the magnet is equal to $2\pi nr \times Cm/r^2 = 2\pi nCm/r$.

Suppose the needle to be deflected through an angle δ from the magnetic meridian NS (see Fig. 6) so as to lie along AB. It is acted upon by a *deflecting couple* due to the equal forces at A and B (along RA and TB respectively), and the arm of this couple is RT, which is equal to $l \cos \delta$, l being the length AB of the needle. Thus the moment of the deflecting couple is

$$2\pi nCm \times l \cos \delta/r.$$

The earth's field acts upon each pole with a force mH[1] in the direction of the arrows: the arm of this *directive couple* is $PQ = l \sin \delta$, and its moment is $mH \times l \sin \delta$.

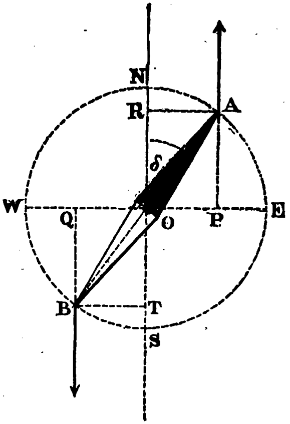

Fig. 6.

For equilibrium these moments must be equal, or

$$2\pi nCm.l \cos \delta/r = mH.l \sin \delta,$$

and $$\therefore C = \frac{r}{2\pi n} \cdot H \tan \delta \quad . \quad . \quad . \quad (6)$$

The quantity $rH/2\pi n$, by which the tangent of the deflection has to be multiplied in order to obtain the current-strength, may be called the *reduction-factor* of the galvanometer: but we shall use this term to denote the value of the quantity k in the equation $C = k . \tan \delta$, where the current C is understood to be expressed in *ampères* instead of C.G.S. units.

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[1] The letter H is the symbol usually adopted for representing the horizontal intensity of the earth's magnetic force.

47. A current of 0·85 ampère flows through a tangent galvanometer consisting of three turns of wire, each of 30 cm. diameter: what is the strength of the field due to the circular current at its centre? If the value of H at the place is 0·17, through what angle will the needle be deflected?

The strength of the field produced by the current is measured by the force which would be experienced by a unit magnetic pole, and this by the preceding paragraph is $2\pi nC/r$. Since an ampère is one-tenth of the C.G.S. unit of current, the required strength of field is $2\pi \times 3 \times 0{\cdot}085/15 = 0{\cdot}1068$. By equation (6)

$$\tan\delta = \frac{2\pi n}{r} \cdot \frac{C}{H} = \frac{2\pi \times 3}{15} \times \frac{0{\cdot}085}{0{\cdot}17} = 0{\cdot}628.$$

Referring to the table of tangents we see that since tan $39^\circ = 0{\cdot}629$, the angular deflection will be nearly 39°.

48. The same current is sent through two concentric circular wires of 1 and 3 decimetres radius respectively, and a small magnet is suspended at their common centre. The currents flow in opposite directions: compare their joint effect upon each pole of the magnet with that produced by a single coil of 2 decimetres radius traversed by the same current.

The force due to the coil of 1 dcm. radius is $f = 2\pi Cm/10$, and that due to the coil of 3 dcm. radius is $f' = 2\pi Cm/30$. Their joint effect (since the forces act in opposite directions) is

$$F = f - f' = 2\pi Cm\left\{\frac{1}{10} - \frac{1}{30}\right\} = 2\pi Cm \times \frac{2}{30}.$$

The force due to the coil of 2 dcm. radius is $F' = 2\pi Cm/20$.

Thus $$F : F' = \frac{2}{30} : \frac{1}{20} = 4 : 3.$$

[This is not the ratio of the *deflections*. If the deflections are δ and δ' respectively, then $\tan\delta : \tan\delta' = 4 : 3$.]

49. Find the strength of a current which produces a deflection of 45° in a tangent galvanometer consisting of a single copper rod bent into a circle of 40 cm. diameter. $[H=0\cdot17.]$

50. A current of 0·04 ampère flows through a circular coil of wire consisting of 3 turns, each 2 dcm. in diameter: what force will be exerted by the current upon a magnetic pole of strength 10 placed at the centre of the coil?

51. A battery is connected up in series with a known resistance R and a tangent galvanometer of resistance G (Fig. 7). The deflection of the galvanometer needle is α: on increasing the known resistance to R' the deflection falls to α'. Show that the internal resistance of the battery is given by the equation

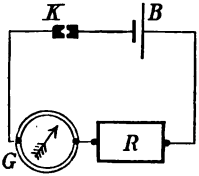

Fig. 7.

$$B=\frac{R'\tan\alpha'-R\tan\alpha}{\tan\alpha-\tan\alpha'}-G.$$

52. Calculate the internal resistance of a Grove's cell from the following data, obtained by the method of Ex. 50:—

Resistance of Galvanometer $=0\cdot102$ ohm.

(1) $R=2$ ohms, $\alpha=53^\circ$.
(2) $R'=5$ ohms, $\alpha'=30^\circ$.

53. What does the equation in Ex. 50 reduce to when the second deflection indicates a current of one-half the original strength?

A current from a battery of 4 Daniell cells was sent through a resistance-box and a tangent galvanometer of negligible resistance. With a resistance of 58 ohms a deflection of 55° was obtained, and when the resistance was increased to 131 ohms the deflection fell to $35\tfrac{1}{2}^\circ$. Find the resistance of the battery, given that $\tan 55^\circ=2\tan 35\tfrac{1}{2}^\circ$.

54. A Minotto cell gave the following results, according to the method of Ex. 50:—

(1) $R = 37$ ohms, $\alpha = 55°$.
(2) $R' = 151$ ohms, $\alpha' = 35\frac{1}{2}°$.
Resistance of Galvanometer $= 36$ ohms.

What was the resistance of the cell?

55. A deflection α is obtained with a tangent galvanometer G connected up as in Fig. 7 with a battery B and a known resistance R. When R is replaced by a wire of unknown resistance the galvanometer deflection is α': show that the value of the unknown resistance is $(\tan\alpha/\tan\alpha')(B+G+R)-(B+G)$.

56. Two single concentric circles of wire are placed with their planes in the magnetic meridian, and a small magnet carrying a mirror is suspended at their common centre. The same current is sent through both circles successively, and the deflections of the spot of light are 40 and 65: find the ratio of the diameters of the circles.

57. The terminals of a battery are connected by a wire 10 ft. long; the wire is wrapped once round the frame of a tangent galvanometer and a certain deflection is produced. The wire is now removed and replaced by another piece of the same wire 50 ft. long. It is found that this second wire has to be coiled three times round the galvanometer frame in order to produce the same deflection. Prove that the resistance of the battery is equal to that of 10 feet of the wire.

58. A tangent galvanometer has two coils, one a thick-wire coil of resistance G, the other a thin-wire coil of resistance G′, and the ratio of the reduction-factors (see p. 215) of the two coils is R. When the first coil is connected up through an unknown resistance with a battery of constant E.M.F. and negligible resistance, a deflection α is produced; and when it is replaced by the second coil the deflection is α'. Prove that the unknown resistance is equal to $(RG - R'G')/(R' - R)$, where $R' = \tan\alpha'/\tan\alpha$.

59. The battery used on a certain telegraph line has a resistance equal to that of 30 miles of the line wire. The current is tested at a station 120 miles from the battery by a galvanometer wound with twenty turns of wire having the same resistance per metre as the line wire. Show that in order to obtain the same deflection at a distance of 270 miles from the battery the number of turns of wire on the galvanometer would have to be doubled. (Neglect the resistance of the return circuit through earth, and assume that the coils of wire on the galvanometer have the same radius throughout.)

60. The current from a battery is sent through a tangent galvanometer, the reduction-factor (p. 215) of which is k, and it produces a deflection α: when an additional resistance r is introduced into the circuit the deflection falls to α'. Show that the E.M.F. of the battery is equal to $kr \,.\, \tan\alpha \,.\, \tan\alpha' / (\tan\alpha - \tan\alpha')$.

61. The poles of a battery of negligible resistance are connected by a tangent galvanometer of resistance G and a wire of unknown resistance. A deflection α is produced: on increasing the resistance of the circuit by a known amount R, the deflection is diminished to β. Show that the resistance of the wire is equal to

$$R \tan\beta/(\tan\alpha - \tan\beta) - G.$$

Electrolysis.—The weight of an element which is set free by electrolysis in one second by a current of unit strength is called the electro-chemical equivalent of that element. In giving the value of the electro-chemical equivalent of an element it is necessary to state whether the current strength is expressed in C.G.S. units or ampères. According to Lord Rayleigh's experiments a current of one ampère deposits 0·001118 gm. of silver, or 0·0003296 gm. of copper per second.

The weight of an element which is set free by a current of strength C in time t is given by the equation

$$W = Cet \quad . \quad . \quad . \quad . \quad . \quad (7)$$

e denoting the electro-chemical equivalent of the element.

Electrolysis provides us with a convenient means of finding the reduction-factor (p. 215) of a tangent galvanometer. Connect up the galvanometer G, as in Fig. 8, with an electrolytic cell or voltameter V and a constant battery B consisting of one or more Daniell cells or accumulators. The voltameter may consist of a pair of copper plates immersed in a beaker containing a saturated solution of copper sulphate: for more accurate work a silver voltameter should be employed. Let W denote the weight of metal deposited in time t by a constant current C which produces a deflection α of the galvanometer needle. If k is the reduction-factor of the galvanometer, $C = k \, . \tan \alpha$: also, by equation (7),

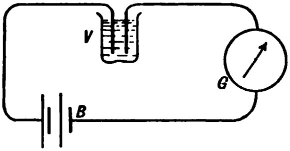

Fig. 8.

$$C = W/et,$$

$$\therefore k \, . \tan \alpha = W/et,$$

and $$k = W/et \, . \tan \alpha.$$

62. How much copper will be deposited by a current of one ampère in an hour?

63. A current of 0·5 ampère is used for preparing pure silver by electrolysis: how long must the current be allowed to flow in order to obtain a deposit of 4 grammes?

64. What is the strength of a current which deposits a milligramme of copper per minute?

65. It is found that a current of 1·868 ampère deposits 1·108 gm. of copper in half an hour: what value does this give for the electro-chemical equivalent of copper?

66. What is the strength of a current which deposits 0·935 gm. of copper in an hour and 10 minutes?

67. A Siemens C_2 dynamo is capable of depositing 6 kilogrammes of copper per hour: what is the strength of the current produced by it?

68. A constant current was sent for half an hour through the thick coils of a Helmholtz galvanometer, producing a deflection of 40°, and also through a copper voltameter in which 1·325 gm. of copper was deposited. What was the reduction-factor of the galvanometer?

69. Assuming the value of the electro-chemical equivalent of silver as given on p. 219, find how much water will be decomposed by a current of 1 ampère in an hour. [Atomic weight of silver = 108, of oxygen = 16, of hydrogen = 1.]

Kirchoff's Laws.—I. In any branching network of wires, the algebraical sum of all the currents which meet at a point is zero, or

$$\Sigma(C) = 0.$$

II. The electromotive force acting around any closed circuit is found by multiplying the resistance of each portion of the circuit into the current which flows through it, and adding together the products thus obtained, or

$$\Sigma(E) = \Sigma(CR).$$

Divided Circuits and Shunts.—Suppose a circuit to divide as at A in Fig. 9, part of the current flowing through a conductor of resistance *r* and part through a conductor of resistance *r′*, reuniting at B; then the two wires are said to be arranged in parallel or in multiple arc, and the resistance of the divided circuit between A and B is equal to the product of the resistances of the two branches divided by their sum.

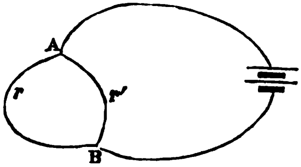

Fig. 9.

For let *e* be the E.M.F. between A and B, *c* the current through the branch r, and c' the current through the branch r'. By Ohm's law

$$c = e/r, \text{ and } c' = e/r' \quad . \quad . \quad . \quad (8)$$

Now apply Kirchoff's first law to the point A. Let C be the current in the main circuit flowing *towards* A; then c and c' flow *away* from A,

$$\therefore C - c - c' = 0,$$

or

$$C = c + c' = e/r + e/r'.$$

The equivalent resistance of the divided circuit is

$$R = \frac{e}{C} = \frac{e}{e/r + e/r'} = \frac{1}{1/r + 1/r'} = \frac{rr'}{r + r'}, \quad . \quad . \quad (6)$$

It is evident from the above equation (8) that the currents in the two branches are inversely proportional to their resistances.

Also, by equation (9),

$$e = CR = C \,.\, rr'/(r + r'),$$

therefore the current in the branch r is

$$c = e/r = C \,.\, r'/(r + r'),$$

and the current in the branch r' is

$$c' = e/r = C \,.\, r/(r + r').$$

By similar reasoning it can easily be proved that the resistance of a multiple arc consisting of n conductors, each offering a resistance r, is $R = r/n$.

When the terminals of a galvanometer are connected directly by a wire, or through a resistance-box, the galvanometer is said to be *shunted*, and the connection is called a *shunt*. If C is the current in the main circuit, G the resistance of the galvanometer, and S that of the shunt, the current through the shunted galvanometer is

$$C_g = C \,.\, \frac{S}{G + S},$$

and the current through the shunt is

$$C_s = C \cdot \frac{G}{G+S}.$$

70. Eight incandescent lamps are arranged in four parallel groups of two each (as in Fig. 10) between two electric light leads. The difference of potential between the leads is 108 volts, and each lamp takes a current of 1·2 ampère. What is the equivalent resistance between the leads?

Since the current through each of the four branches is 1·2 ampère, the resistance of each group (of two lamps in series) is 108/1·2 = 90 ohms, or the resistance per lamp is 45 ohms. The joint resistance of the four groups in parallel is 90/4 = 22·5 ohms. (Here we assume that the resistance of the leads is very small, and that the lamps are placed close together, so that the E.M.F. is constant.)

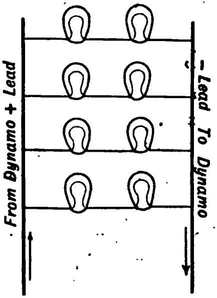

Fig. 10.

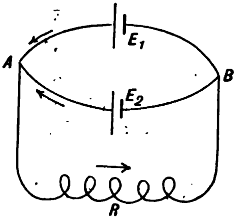

Fig. 11.

71. Two batteries of E.M.F.'s E_1 and E_2, and resistances r_1 and r_2, are arranged in parallel so that they tend to send currents in the same direction through an external resistance R (Fig. 11). What is the strength of the current in the external circuit?

Let C be the current through the external resistance, c_1 the current through the battery E_1, and c_2 the current through the battery E_2.

By Kirchoff's second law, we have in the circuit E_1CR

$$c_1r_1 + CR = E_1,$$

and in the circuit E_2CR

$$c_2r_2 + CR = E_2.$$

By Kirchoff's first law $C = c_1 + c_2$,

$$\therefore C = (E_1 - CR)/r_1 + (E_2 - CR)/r_2,$$

$$Cr_1r_2 = E_1r_2 + E_2r_1 - C(r_1R + r_2R),$$

and $$C = (E_1r_2 + E_2r_1)/(r_1r_2 + r_1R + r_2R).$$

72. A 1-ohm coil is tested and found to have a resistance of 1·004 ohm. What length of platinoid wire having a resistance of $16\frac{2}{3}$ ohms per metre must be wound in multiple arc between the terminals in order to make it a correct ohm?

If x is the required resistance of the platinoid wire, we have, by equation (9),

$$1 = 1{\cdot}004\,x/(1{\cdot}004 + x),$$

$$\therefore 1{\cdot}004 + x = 1{\cdot}004\,x,$$

or $$1{\cdot}004 = 0{\cdot}004\,x, \text{ and } x = 251.$$

The shunt wire must therefore have a resistance of 251 ohms, and the required length is $251 \times 3/50 = 15{\cdot}06$ metres.

73. A uniform wire is bent into the form of a square: find the resistance between two opposite corners in terms of the resistance of one of the sides.

74. What is the resistance between the extremities of one of the sides in the preceding example?

75. Twelve incandescent lamps are arranged in parallel between two electric light leads. The difference of potential between the leads is 99 volts, and each lamp takes a current of 0·75 ampère: what is the equivalent resistance between the leads?

76. A divided circuit consists of two wires of the same material whose lengths are l and l', and cross-sections s and s' respectively: compare the currents in the two wires.

77. The resistance of a circuit, including that of the battery, is 18 ohms. What change will be produced in the current through the battery if two points of the circuit, between which the resistance is 12 ohms, are connected by a wire of 4 ohms resistance?

78. A standard coil having a resistance of one legal ohm is connected in multiple arc with a resistance-box, and you are required to take plugs out of the box until the joint resistance of the two is exactly equal to a B.A. unit (see p. 6). A legal ohm is equal to 1·0112 B.A. units: how much resistance must be taken out?

79. Three wires whose resistance are r_1, r_2 and r_3 respectively are connected in multiple arc: prove that the resistance of the multiple arc is $r_1r_2r_3/(r_1r_2+r_2r_3+r_3r_1)$.

80. Two points A and B are connected by three wires whose resistances are 1, 3 and 6 ohms respectively: find the total current which passes through the multiple arc when the difference of potential between A and B is 3 volts.

81. A divided circuit consists of two equal and similar wires: what alteration will be produced in its resistance by making the wires touch so that a point one-quarter the length from an end of the one wire is in contact with a point three-quarters the length from the corresponding end of the other wire?

82. An equilateral triangle, one foot in the side, is made of wire of uniform material but of unequal thickness; the base being made of wire 0·8 mm. thick, and the sides being 1·0 mm. and 1·2 mm. respectively: find the resistance between the two extremities of the base, taking the resistance of an inch of the base wire as unity.

83. A skeleton cube is made up of twelve uniform wires: prove that the resistance between diagonally opposite corners of the cube is equal to five-sixths of the resistance of one of the wires.

84. A tangent galvanometer of 10 ohms resistance is shunted by a shunt of resistance 0·64 ohm. A steady current is passed for an hour through the shunted galvanometer, and also through a silver voltameter in which 0·532 gm. of silver is deposited, the deflection of the galvanometer being 52°. Express the strength of the current in ampères, and calculate the reduction factor of the galvanometer.

If C be the strength of the current, then, by the equation $W = Cet$, we have

$$0{\cdot}532 = C \times 0{\cdot}001118 \times 3600 = C \times 4{\cdot}025,$$

$$\therefore C = 0{\cdot}532/4{\cdot}025 = 0{\cdot}1321 \text{ ampère.}$$

The current passing through the galvanometer is

$$C_g = C \times S/(G+S),$$

where S is the resistance of the shunt, and G that of the galvanometer. But if k be the required reduction-factor, we have also

$$C_g = k \,.\, \tan \delta,$$

and $$\therefore k = C_g/\tan\delta = C \,.\, S/(G+S)\tan\delta,$$

$$= 0{\cdot}1321 \times 0{\cdot}64/10{\cdot}64 \times 1{\cdot}28 = 0{\cdot}00621.$$

85. A battery B of low resistance is joined up in circuit with a resistance-box R and a shunted galvanometer as in Fig. 12. After noting the deflection the shunt is removed, and it is found that the resistance R has to be increased to a higher value R′ in order to reduce the galvanometer deflection to its original value. Prove that, if the internal resistance of the battery be neglected, the resistance of the galvanometer is equal to S(R′ − R)/R.

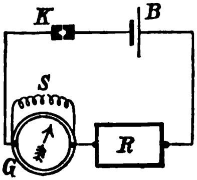

Fig. 12.

86. A battery of resistance B is connected up with a galvanometer of resistance G. The galvanometer is then shunted by a wire of resistance S. Compare the currents produced by the battery in the two cases, and show that a *compensating resistance* equal to $G^2/(G+S)$ must be introduced into the circuit if it is required to keep the current unaltered on shunting the galvanometer.

87. What shunt is required to reduce to one-hundredth the sensitiveness of a galvanometer of 396 ohms resistance?

88. A galvanometer of 45 ohms resistance is shunted by a shunt of 5 ohms. Find the resistance of the shunted galvanometer and the current which flows through it when a difference of potential of 22·5 volts is maintained between its terminals.

89. A battery of 20 ohms resistance is joined up in circuit with a galvanometer of 10 ohms resistance. The galvanometer is then shunted by a wire of the same resistance as its own: compare the currents produced by the battery in the two cases.

90. In the preceding example determine the ratio between the currents which flow through the galvanometer before and after it is shunted.

91. In measuring the resistance of a battery by Thomson's method, it is joined up in circuit with a galvanometer G and a resistance-box R as shown in Fig. 13, the battery itself being shunted by a wire of resistance S. On removing the shunt a larger current passes through the galvanometer; but by increasing the resistance R to some higher value R′ the same deflection can be obtained. Prove that the resistance of the battery is given by the equation

$$B = S(R' - R)/(R + G).$$

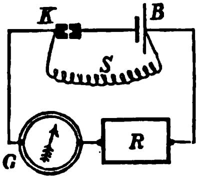

Fig. 13.

92. It is found that on shunting a certain galvanometer the current through it is reduced to one-half, while the battery current is doubled: show that the resistance of the galvanometer is double that of the battery, and three times that of the shunt.

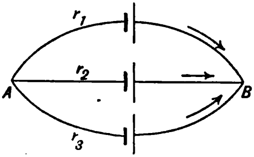

Fig. 14.

93. The like poles of three cells are connected in multiple arc (Fig. 14) by wires of inappreciable resistance, so that their E.M.F.'s all act towards the same point B. Prove that the currents through the several branches are given by equations of the form

$$c_1 = \frac{e_1(r_2 + r_3) - e_2r_3 - e_3r_1}{r_1r_2 + r_2r_3 + r_3r_1}, \text{ etc.,}$$

where e_1, e_2, and e_3 are the E.M.F.'s of the three cells, and r_1, r_2, r_3 the resistances of the three branches respectively.

[Apply Kirchoff's laws. *Some* definite directions—say those indicated in the figure—must be *assumed* for the currents, but it is evident that all three currents cannot really flow from A to B.]

94. Two equal cells when connected in series with a given wire produce a current of 0·28 ampère; when connected in multiple arc through the same wire the current produced is 0·2 ampère. Prove that the resistance of the wire is three times that of either cell.

95. Two equal cells are connected (1) in series, (2) in parallel, the external resistance in both cases being the same. The currents produced are in the ratio of 4 to 3: prove that the resistance of the wire is two and a half times that of either cell.

96. You are provided with a delicate galvanometer of 48 ohms resistance, and are required to measure by means of it a current, the strength of which is supposed to be 2·5 ampères: if the largest current that can safely be sent through the galvanometer is 0·1 ampère, what resistance would you supply as a shunt?

If the deflection of the shunted galvanometer indicates a current of 0·085 ampère, what is the current in the main circuit?

97. If the points A and B in Fig. 14 are connected by a wire of resistance R, what currents will flow through the wire and through each of the cells?

98. You are required to replace the two cells in Ex. 71 (Fig. 11) by a single battery, which will send the same current through the external resistance R. Prove that this battery must have an E.M.F. equal to $(e_1r_2+e_2r_1)/(r_1+r_2)$, and that its internal resistance must be $r_1r_2/(r_1+r_2)$.

Arrangement of Cells for Maximum Current.—When the internal resistance of the cells of a battery is so small that it can be neglected in comparison with the external resistance of the circuit, the current produced is approximately proportional to the number of cells employed. But if the external resistance is small compared with that of a single cell, the current is only slightly increased by adding on cells in series: for although every additional cell increases the E.M.F., it also increases the resistance in nearly the same proportion. A stronger current can be obtained by arranging the cells either partly or wholly in parallel (as in Fig. 11), so as to diminish the internal resistance.

When the external resistance and the number of cells are given, the maximum current is obtained by grouping the cells in such a way that *the internal resistance of the battery is (as nearly as possible) equal to the external resistance of the circuit.*

For let R be the external resistance, N the number of cells, e the E.M.F. and r the resistance of each cell. Suppose the cells to be arranged in m rows, each row consisting of n cells in series, then

$$m\times n=\text{N} \quad . \quad . \quad . \quad . \quad (a)$$

The E.M.F. of the battery is the same as that of each row, and is equal to ne. Each row consisting of n cells has a resistance nr: the equivalent resistance of the m rows arranged in parallel is nr/m.

Thus, by Ohm's law, the current produced is

$$C=\frac{ne}{\frac{nr}{m}+R}=\frac{e}{\frac{r}{m}+\frac{R}{n}}.$$

Now e is constant, and therefore C is a maximum when $r/m+R/n$ is a minimum. Observe also that the product rR/mn is constant, because r and R are given, and mn is a constant, being equal to the total number of cells. But if the product of two factors is constant it can be shown that their sum is a minimum when they are equal to one another.

Thus if pq is constant, $p+q$ is a minimum when $p=q$. For $p+q=(\sqrt{p}-\sqrt{q})^2+2\sqrt{pq}$, and, since the last term is constant, $p+q$ is a minimum when $(\sqrt{p}-\sqrt{q})^2$ is a minimum. Now the minimum value of a square is zero: hence $p+q$ is a minimum when $\sqrt{p}-\sqrt{q}=0$, *i.e.* when $p=q$.

If then we wish to give C its maximum value we must make

$$\frac{r}{m}=\frac{R}{n}, \text{ or } \frac{nr}{m}=R \quad . \quad . \quad . \quad . \quad (b)$$

But nr/m is the internal resistance, and R is the external resistance, so that the maximum current is obtained when the resistance of the battery is made equal to that of the external circuit.

99. What is the best way of arranging a battery of 18 cells, each having a resistance of 1·8 ohm, so as to send the largest current through an external resistance of 1 ohm?

By equation (a),

$$m \times n = 18,$$

and by the condition for maximum current [equation (b)],

$$1{\cdot}8n/m=1.$$

Thus $n^2=10$, and the nearest integer to $\sqrt{10}$ is 3. It follows that $m=18/3=6$, and that the cells must be arranged in 6 rows of 3 each.

100. The current from a battery of 4 equal cells is sent through a tangent galvanometer, the resistance of which, together with the attached wires, is exactly equal to that of a single cell. Show that the galvanometer deflection will be the same whether the cells are arranged all abreast or all in series. What is the best arrangement that could be adopted, and what would be the corresponding deflection?

101. You are supplied with 12 exactly similar cells, the internal resistance of each of which is one-fourth of the external resistance of the circuit: how would you group the cells so as to obtain the maximum current?

102. How would you arrange a battery of 12 cells, each of 0·6 ohm internal resistance, so as to send the strongest current through an electromagnet of resistance 0·7 ohm?

103. You have a battery of 48 Daniell cells, each of 6 ohms internal resistance, and are required to send the largest possible current through an external resistance of 15 ohms: how would you group the cells? Find also the current produced and the difference of potential between the poles of the battery, assuming that the E.M.F. of a Daniell cell is 1·07 volt.

104. If there are 18 cells in a battery, each of resistance 1·5 ohm, how can they best be arranged so as to send a strong current through an external circuit of 3·5 ohms resistance?

Power and Heating Effect of a Current.—Suppose a current C to flow through a portion of a circuit, the ends of which are at a difference of potential E. The total quantity of electricity which passes any point in the circuit during a time t is $Q = Ct$. The work done by the current during the same time is measured by the product of the quantity of electricity thus transferred into the difference of potential (p. 5), or

$$W = QE = CEt. \qquad . \quad . \quad . \quad (10)$$

The *power* or *activity* of the current is measured by its rate of doing work, *i.e.*

$$P = QE/t = CE \qquad . \quad . \quad . \quad (11)$$

If C and E are expressed in C.G.S. units the above equations give the work in ergs and the power in ergs per second. Since the ampère = 10^{-1} C.G.S. units of current, and the volt = 10^8 C.G.S. units of E.M.F., a current of one ampère flowing through a difference of potential of one volt does work at the rate of $10^{-1} \times 10^8 = 10^7$ ergs per second. This is adopted as the practical unit of power, and is called a **Watt.** Thus the rate at which work is done in a circuit is measured in watts by the product of the current in ampères into the E.M.F. in volts. We have already shown (p. 14) that a horse-power is approximately equal to 746 watts.

If the portion of the circuit under consideration contains no other source of E.M.F. besides that which produces the current, it follows from Ohm's law that the above equations may be written in the form

$$W = C^2Rt \quad . \quad . \quad . \quad . \quad (12)$$

and

$$P = C^2R \quad . \quad . \quad . \quad . \quad (13)$$

On the other hand, if a voltameter, or an electromotor in motion, is included in the circuit, the current is diminished by an amount corresponding to the counter electromotive force thus set up, and Ohm's law cannot be applied; but in all cases CEt measures the work done, and CE the power.

When a current flows through a metallic conductor without doing any work beyond overcoming the resistance of the conductor, its energy is entirely converted into heat, which goes to raise the temperature of the conductor. If H is the amount of heat thus produced, its equivalent in ergs is JH, and therefore by equation (12)

$$JH = C^2Rt \quad . \quad . \quad . \quad . \quad (14)$$

This equation expresses what is known as *Joule's law.* Here again we have supposed that current and resistance are both expressed in C.G.S. units. If C is expressed in ampères, its equivalent in C.G.S. units is $C \times 10^{-1}$; and if R is expressed in ohms its equivalent in C.G.S. units is $R \times 10^9$. The numerical value of J (Joule's equivalent; see p. 140) is $4 \cdot 2 \times 10^7$: thus the amount of heat generated in time t by a current of C ampères flowing through a resistance of R ohms is

$$H = (C \times 10^{-1})^2 \times (R \times 10^9)t/4 \cdot 2 \times 10^7,$$

or

$$H = C^2Rt/4 \cdot 2 \quad . \quad . \quad . \quad (15)$$

105. The output of a dynamo is frequently expressed in "units" of 1000 watts each: thus a "four-unit" machine produces 4000 watts. How many 16-candle-power lamps, each requiring 3·5 watts per candle, can be run by such a dynamo? If the lamps used are 50-volt lamps, what current does each take?

Each lamp absorbs $16 \times 3 \cdot 5 = 56$ watts, and if n is the required number of lamps, $56n = 4000$, and $n = 71 \cdot 4$. If 71 lamps are used, and the dynamo is run at its normal speed, the lamps will be somewhat too bright: if 72 are used, they will be somewhat dull.

The number of watts absorbed by each lamp is equal to the product of the current in ampères into the E.M.F. in volts. If C is the current taken by each lamp,

$$C \times 50 = 56,$$

and

$$C = 56/50 = 1 \cdot 02 \text{ ampère}.$$

106. The poles of a battery, of internal resistance B, are connected successively by two wires, whose resistances are R_1 and R_2 respectively: if B is a mean proportional between R_1 and R_2, show that the quantities of heat developed in equal times in each of the two wires are the same.

Let E denote the E.M.F. of the battery. The currents produced in the first and second cases respectively are

$$C_1 = E/(B + R_1), \text{ and } C_2 = E/(B + R_2).$$

The quantity of heat developed in time t in the wire of resistance R_1 is

$$H_1 = \frac{1}{J}\cdot C_1^2R_1t = \frac{1}{J}\left(\frac{E}{B+R_1}\right)^2 R_1t.$$

Substituting for B its value $\sqrt{R_1R_2}$, this gives

$$H_1 = \frac{R_1t}{J}\cdot\frac{E^2}{R_1R_2+2R_1\sqrt{R_1R_2}+R_1^2} = \frac{t}{J}\cdot\frac{E^2}{R_1+2\sqrt{R_1R_2}+R_2}$$

Since this expression is symmetrical with respect to R_1 and R_2, it is clear that it also represents the heat developed in the same time in the wire R_2.

107. The current from a single secondary cell was sent through the thick coils of a Helmholtz galvanometer, and also through a coil of wire of 1 ohm resistance immersed in 100 grammes of water. In 40 minutes a rise in temperature of 15°·8 was produced, and the mean deflection of the galvanometer needle was 32°. Calculate the strength of the current, and the reduction-factor of the galvanometer.

The amount of heat produced is $H = 100 \times 15\cdot8$. The resistance of the coil is 10^9 C.G.S. units, and, according to equation (14), p. 232, the value of the current is given by the equation

$$(4\cdot2 \times 10^7) \times 100 \times 15\cdot8 = C^2 \times 10^9 \times 40 \times 60,$$

$$\therefore\ C^2 = 4\cdot2 \times 15\cdot8/2400 = 0\cdot02765,$$

and $$C = 0\cdot1663.$$

Thus the current strength is 0·1663 C.G.S. unit or 1·663 ampère.

The reduction-factor (k) is defined by the relation $C = k.\tan\delta$, and since $\tan 32° = 0\cdot625$, we have

$$k = C/\tan\delta = 1\cdot663/0\cdot625 = 2\cdot661.$$

[This is the factor for reducing readings to ampères; if the current is required in C.G.S. units the corresponding reduction factor will be 0·2661.]

108. Compare the amounts of power required to send a current of constant strength through two wires of resistances r_1 and r_2, (1) when they are arranged in series, and (2) when they are arranged in parallel.

109. If, instead of a constant current, a constant E.M.F. were used in the preceding problem, what would be the ratio between the amounts of power absorbed in the two cases?

110. What horse-power is required to maintain a current of 4 ampères through a resistance of 37·3 ohms?

111. An arc lamp takes a current of 9·75 ampères, and the E.M.F. between its terminals is 50 volts: what power does it absorb?

112. In the Provisional Orders of the Board of Trade the unit of electrical supply is defined as 1000 *ampères flowing for one hour under a pressure of one volt.* Prove that this is equal to 1·34 horse-power working for one hour.

113. A house is lit by 23 Bernstein lamps of 20 candle-power, each lamp taking a current of 10 ampères, and requiring an E.M.F. of 10 volts. The total length of the conducting wires is 440 yards, and they are made of No. 12 B.W.G. copper wire having a resistance of 4·6 ohms per mile. Show that the amount of energy lost in the conductors is one-twentieth of that used in the lamps.

114. The poles of two batteries, each consisting of 10 cells, are connected by thick copper wires of inappreciable resistance; the plates of the one battery are three times as large as the plates in the other, but the cells are similar in all other respects. Compare the intensities of the currents, and the amounts of heat generated in the same time in the two batteries.

115. A current of 1·4 ampère is allowed to flow for half an hour through a coil of wire of 5 ohms resistance immersed in 250 grammes of water. What elevation of temperature will be produced by it?

116. An incandescent lamp of 16 candle-power takes a current of 0·75 ampère with a difference of potential of 50 volts between its terminals: find the number of watts per candle-power absorbed by the lamp, and the amount of heat generated in it in an hour.

117. In order to determine the strength of a current it was made to pass through a coil of wire of 5 ohms resistance placed in a calorimeter: a steady stream of water was kept flowing through the calorimeter at the rate of 15 c.c. per minute, and the heating effect of the current was such that the water was 4° warmer on leaving the calorimeter than it was on entering. Find the strength of the current.

118. The amount of heat generated per second by a current of c ampères in a copper wire 1 metre long and d millimetres in diameter is given by the equation $H = k \, . \, (c/d)^2$. Calculate the value of the constant (k) in this equation, taking the specific resistance of copper in C.G.S. units to be 1600.

119. When a current divides between two wires arranged in multiple arc, the current in each branch is inversely proportional to its resistance (p. 222): prove that the total amount of heat generated in the two wires is less than it would be if the current were to divide between them in any other proportion.

Efficiency of Dynamos and Motors.—In the following examples the term efficiency will be used to denote the *efficiency of conversion*, which is the ratio between the electrical power developed and the horse-power required to drive the dynamo. A portion of the power developed in the circuit of a dynamo is always absorbed in overcoming the resistance of the dynamo itself; and the ratio of the power developed in the external circuit to the total power is sometimes called the *electrical efficiency* of the system. This is equal to $R/(R+r)$, where r is the resistance of the dynamo and R that of the external circuit (supposing the latter to contain no opposing E.M.F.) What we have called the efficiency (*i.e.* the efficiency of conversion) depends upon the construction of the dynamo itself, whereas the electrical efficiency depends not only upon the internal resistance of the dynamo but also upon that of the external circuit

It is therefore clear that we should distinguish between the *gross* and *nett* efficiency of conversion. The gross efficiency is the ratio of the total electrical horse-power to the horse-power required to drive the dynamo: the nett efficiency is the ratio between the electrical horse-power in the external circuit and the horse-power absorbed by the dynamo. This latter quantity measures the economy of the system as a whole, and is commonly called the *commercial efficiency*.

The efficiency of a motor is the ratio between the horse-power developed by the motor and the electrical horse-power absorbed by it.

120. Find the useful commercial efficiency of an Edison No. 5 dynamo from the following tests made at the Franklin Institute (Philadelphia):—

E.M.F. at terminals of dynamo . .	125·2 volts.
Current in external circuit . . .	100·9 ampères.
Resistance of external circuit . .	1·241 ohm.
Power applied	18·89 H.-P.

The power developed in the external circuit is $CE = 100{\cdot}9 \times 125{\cdot}2 = 12633$ watts, or $12633/746 = 16{\cdot}94$ H.-P.

[The power can also be calculated from the given current and resistance: this gives the same result, for $C^2R = (100{\cdot}9)^2 \times 1{\cdot}241 = 12634$ watts.]

The commercial efficiency is $16{\cdot}94/18{\cdot}89 = 0{\cdot}8968$, or 89·68 per cent.

121. A Weston No. 7 M dynamo tested at the Franklin Institute was found to give a current of 123·6 ampères through an external resistance of 1·224 ohm, the potential difference at the terminals of the dynamo being 151·2 volts. The power absorbed was 28 H.-P.: show that the commercial efficiency of the dynamo was 89·5 per cent.

122. A petroleum engine delivering 7 H.-P. was coupled up with a dynamo used for charging secondary batteries. The average current was 77 ampères, with an E.M.F. of 58 volts: find the electrical horse-power and the efficiency of the dynamo.

123. A dynamo driven by a gas-engine of 2 H.-P. (actual) is used for supplying a current to a circuit whose resistance (including that of the dynamo) is 3·357 ohms. Show that if the efficiency of the dynamo is 90 per cent, it will produce a current of 20 ampères.

124. The total weight of a tram-car driven by an electromotor (including the weight of the passengers) is 5 tons. The current for the motor is derived from a battery of accumulators, the average E.M.F. of which is 110 volts during the discharge. Assuming the motor to have an efficiency of 70 per cent, find the rate at which the car will run on the level when the discharge current is 18·65 ampères, the resistances due to friction, etc., being at the rate of 28 lbs. per ton.

The power developed by the motor is $18{\cdot}65 \times 110 \times 0{\cdot}7$ watts, which is equivalent to $18{\cdot}65 \times 110 \times 0{\cdot}7/746$ H.-P. Thus the motor does work at the rate of $18{\cdot}65 \times 110 \times 0{\cdot}7 \times 550/746$ foot-pounds per second. Suppose that it gives the car a velocity of x feet per second: the work done by it per second will be $5 \times 28 \times x$ ft.-lbs.

Equating these two expressions, we have

$$x = 18{\cdot}65 \times 110 \times 0{\cdot}7/746 \times 5 \times 28 = 7{\cdot}563.$$

Thus the car will run at the rate of 7·563 ft. per sec., or 5·16 miles per hour.

125. Two trials of an Immisch motor gave the following results:—

	Exp. I.	Exp. II.
Current . .	31·5 ampères.	31 ampères.
E.M.F. . .	134 volts.	147 volts.
Power developed	4·7 H.-P.	5·2 H.-P.

Show that the mean efficiency of the motor was 0·84.

126. A smaller motor of the same kind was tested with a current of 25·5 ampères and an E.M.F. of 40 volts, and was found to develop 0·98 H.-P. Show that its efficiency was 0·72, and that the electrical horse-power absorbed by it was 1·37.

127. The following results were obtained with an Ayrton and Perry motor, running at 2240 revolutions per minute:—

Current = 33 ampères.
E.M.F. = 32·5 volts.
Brake horse-power = 0·4586.

What was the efficiency of the motor?

128. An electromotor which is required to develop 2 H.-P. is to be introduced into an arc lighting circuit traversed by a constant current of 15 ampères. Assuming the motor to have an efficiency of 80 per cent, show that the power absorbed by it will be 1865 watts, and that the E.M.F. at its terminals will be $124\cdot\dot{3}$ volts.

EXAMINATION QUESTIONS.

129. Explain the terms current, electromotive force, and resistance.

What is the current in a circuit which consists of a battery of 10 similar cells arranged in series and a wire of 24 ohms resistance? [Each of the cells has an electromotive force of 1·8 volt, and an internal resistance of ·3 of an ohm.] Matric. 1888.

130. Explain the meaning of the term "electrical resistance," and point out the physical laws upon which the scientific value of the term depends.

The difference of potential between the terminals of a voltaic cell when the circuit is not closed is 2 volts. When it is closed through an external resistance of 2·4 ohms, this difference of potential is reduced to 1·5 volt. Calculate the internal resistance of the cell.

Oxford (Prel. Sc.) 1888.

[See Ex. 2, p. 206. The resistance of the cell is 0·8 ohm.]

131. The resistance of a mile of telegraph wire, whose diameter is 0·2 inch, is 8·96 ohms. What must be the thickness of a wire of which 350 yards shall have 2 ohms resistance? Camb. B. A. 1881.

132. If the conductivity of copper is to that of aluminium as 2·3 : 1, and their specific gravities as 3·3 : 1, show what would be the ratio of the weights of wires of these metals, which, for the same length, offered the same resistance. Ind. C. S. 1882.

133. Define the resistance of a conductor, and describe some method of measuring it. The resistance of a column of mercury 106 cm. in length and 1 sq. mm. in section at 0° C. being defined as 1 ohm, find the resistance of a column 2 metres long and 2·5 sq. mm. in section at a temperature of 15° C. Mercury increases in resistance by ·085 per cent per 1° C.

Int. Sc. Honours 1887.

134. Three galvanic cells, *A*, *B*, *C*, whose respective electromotive forces and resistances are as follows, namely—

	A	B	C
Electromotive force .	1·07	1·54	1·9 volt
Resistance . . .	·72	2·3	·1 ohm

are connected in series, and the circuit is completed by a wire of resistance 5·9. Determine the strength of the current produced. If the cell *B* were removed and replaced in the circuit, with its terminals inverted, what would be the strength of the current? Int. Sc. 1885.

135. Explain Ohm's law as applied to a restricted portion of a circuit. The difference of potential between the poles of a battery (of 1·2 ohms internal resistance) is 6 volts when the poles are insulated, and 4·5 volts when they are joined by a wire: what is the resistance of the wire? Int. Sc. 1886.

[See Ex. 2. The resistance of the wire is 3·6 ohms.]

136. A battery of 4 ohms internal resistance is sending a current through an external resistance of 6 ohms.

The terminals of the battery are connected to the electrodes of a reflecting electrometer, and the deflection on the scale is 100 divisions. What will the deflection be, when, everything else remaining the same, the external circuit is broken? Int. Sc. 1886.

137. Describe some form of Thomson's Quadrant Electrometer.

The two poles of a battery are connected to the electrodes of a quadrant electrometer, and readings taken (1) when there is no circuit, (2) when the circuit is completed by a wire of 6 ohms resistance. The mean readings in the two cases being 200 and 147 respectively, determine the resistance of the battery. Prel. Sc. 1888.

138. A battery of Daniell's cells, arranged two abreast, is required to maintain a current of 3 ampères through an external resistance of 2 ohms. How many cells will be required, the E.M.F. of each being 1·2 volt, and the internal resistance of each 0·4 ohm? Prel. Sc. 1887.

139. A battery is connected by short thick wires to a galvanometer, and the deflection noted. The galvanometer is then shunted with one-third of its own resistance, and on connecting again with the battery the current through the galvanometer is observed to have half its former value. Show that the resistance of the battery is half that of the galvanometer. N. S. Tripos. 1886.

140. The resistance of a shunted galvanometer is 85 ohms, that of the shunt being 100 ohms. A certain deflection of the galvanometer needle is obtained when the resistance in the rest of the circuit is 2000 ohms. Find what additional resistance must be inserted that the galvanometer deflection may remain the same when the shunt is removed. B. Sc. 1887.

141. Define the magnetic moment of a magnet, and state the laws which express the direction and magnitude of the force exerted by an electric current passing along a circular wire upon a magnetic pole at the centre of the circle.

A battery-cell of constant electromotive force and negligible resistance is connected up with two galvanometers *A* and *B*, arranged first in series and secondly in multiple arc. In the first case the deflection of *A* is 25 and that of *B* 46; and in the second case that of *A* is 43. Find the deflection of *B*, and compare the constants and resistances of the two galvanometers, assuming that the currents in each case are proportional to the deflections.
N. S. Tripos. 1884.

142. Describe the action of an electric current on a magnet, and explain how it is made use of in a galvanometer.

There are 25 turns of wire in a galvanometer coil, the mean radius of which is 150 cm. Assuming the value of H to be 0·18, find the current which will deflect a magnet placed at the centre of the coil 45°. If the resistance of the circuit, including the battery, be 3 ohms, find the electromotive force required to produce the current.
Int. Sc. 1886.

[See p. 214. The current is equal to 0·172 C.G.S. unit or 1·72 ampère, and the required E.M.F. is 172 × 3 = 5·16 volts.]

143. What is meant by the reduction factor of a galvanometer?

A sine galvanometer consists of a single coil of 49 turns of wire, the mean radius of which is 20 cm. A current of ·08 ampère causes such a deflection that the coil has to be turned through 45° degrees to bring the needle to its original position with regard to the coil. Determine the reduction factor of the galvanometer, and the horizontal component of the earth's magnetic intensity.
Int. Sc. Honours 1887.

144. The terminals of a galvanometer (resistance = G) are connected with a cell (E.M.F. = E_1) and resistance R_1, and at the same time with another cell (E.M.F = E_2) and resistance R_2. Determine the current through the galvanometer in terms of the data, and show that by suitably adjusting the resistances R_1 and R_2 a comparison of the electromotive forces of the cells may be made.

Int. Sc. Honours 1885.

145. State the laws of electrolysis.

A copper voltameter and an acidulated water voltameter are inserted in the same voltaic circuit; and it is found that 1·5 gramme of hydrogen is given off while 47·625 grammes of copper are deposited on the copper cathode. Calculate the atomic weight of copper. [Copper is divalent.] Oxford (Prel. Sc.) 1887.

146. Describe a series of experiments to prove Ohm's law. A current passes from *A* to *D* through a circuit composed as follows: Between *A* and *B* is a resistance of 1 ohm, between *B* and *D* an unknown small resistance; *A* is joined to *C* by a resistance of 2 ohms, and *B* to *C* by one of 99 ohms. The terminals of an electrometer (or of a high-resistance galvanometer) are joined alternately to *A* and *C* and to *B* and *D*, and the deflections are the same in the two cases. Find the value of the unknown small resistance. B. Sc. 1886.

Between A and B there is a divided circuit, one branch of which (AB) has a resistance of 1 ohm, while the other branch (ACB) has a resistance of 101 ohms. If C is the total current, then the current through ACB is C/102, and the potential difference between the points A and C is, by Ohm's law, equal to $2 \times C/102 = C/51$. The potential difference between the points B and D is equal to C*r*, where *r* is the unknown small resistance. Since they are equal, it follows that the value of the unknown resistance is $\frac{1}{51}$ ohm.

147. Two points, *A* and *B*, are connected by three wires, *APB*, *AQB*, *ARB*, whose resistances are 1, 2, and 3 ohms respectively, and *A* is also joined to *R*, the middle point of *ARB*, by a wire *ASR* of 2 ohms resistance. How much of the total current flowing from *A* to *B* passes through each of the two branches between *A* and *R*? B. Sc. 1887.

148. A battery is connected in circuit with one coil of a differential galvanometer, shunted by a wire of resistance S, and the deflection of the needle observed. The battery is then connected with both coils of the galvanometer in series, shunted by a wire of resistance 2S, and resistance is introduced into the circuit until the galvanometer indication is the same as before. Show how the internal resistance of the battery may be obtained from these two observations. B. Sc. Honours 1885.

The proportion of the total current which passes through the shunted galvanometer is the same—$S/(G \times S)$—in both cases, if the resistance of the two coils is the same. Assuming further that with the same current the two coils produce the same effect on the needle, it follows that the current through the galvanometer in the first experiment is twice as strong as in the second. From this it can easily be proved that the internal resistance of the battery is equal to the additional resistance introduced into the circuit.

149. What measurements would you make to test Ohm's law? A battery of E.M.F. 10 volts is working in a circuit whose resistance is 100 ohms. How much work is done by the battery in an hour? How is the energy required for this work produced, and what becomes of it? Prel. Sc. 1888.

150. State Joule's law of the heat developed in wires by electric currents.

A current is passed through a coil of fine wire of 5 ohms resistance, immersed in a vessel containing 100 grammes of water, and the same current also passes through a coil of 4 ohms resistance, immersed in a vessel containing 100 grammes of alcohol. The water rises 2° while the alcohol rises 2·5° in the same time. Find the specific heat of alcohol on the assumption that the heat absorbed by the vessels may be neglected, and the current in both cases merely passes through the wire, and not through the liquids. Vict. Int. 1885.

151. A current passes through two wires arranged in parallel arc. The first wire is of platinum, 15 cm. long and 1 mm. in diameter; the second of German-silver, 20 cm. long and 0·5 mm. in diameter. Compare the quantities of heat developed in the wires in a given time, the relative conductivities of platinum and German-silver being 70 and 33. Prel. Sc. 1886.

152. The resistance of a copper wire, through which an electric current of unknown strength is flowing, is 3·2 ohms, and the difference of potential between its extremities is 2·5 volts. If this wire be immersed in 120 grammes of water, determine the temperature through which the water will be raised in five minutes, assuming that the water absorbs all the heat generated. [$J = 41{\cdot}55 \times 10^6$ ergs.] Int. Sc. Honours 1886.

[See pp. 231-233. The amount of heat generated is given by the equation $JH = E^2t/R$, where $E = (2{\cdot}5) \times 10^8$ and $R = 3{\cdot}2 \times 10^9$. The rise in temperature is $1^\circ{\cdot}177$.]

153. The resistance of an incandescent lamp is 40 ohms, and the difference of potential between its two terminals is 45 volts. Determine the heat produced in it per hour.

[Mechanical equivalent of heat $= 4{\cdot}2 \times 10^7$ ergs.
1 volt $= 10^8$ units of electromotive force.
1 ohm $= 10^9$ units of resistance.]
Int. Sc. Honours 1885.

154. A metal disc revolves in a magnetic field about an axis through its centre and perpendicular to its plane. Determine the electromotive force between centre and circumference. Int. Sc. Honours. 1885.

155. Ninety incandescent lamps are placed in parallel circuit, and a current of 40 ampères is distributed between them, the E.M.F. between the terminals being 120 volts. The resistance of the conductors is ·2 ohm, and that of the dynamo (series) is ·25 ohm, and the insulation resistance between the flow and return conductors is 1000 ohms. What horse-power will be required for electrical work, and how much will be wasted? (1 H.-P. = 746 watts.) Ind. C. S. 1885.

156. A dynamo running at 100 volts is employed to drive a motor. The whole resistance of the circuit, including the dynamo and motor, is only half an ohm, but the whole current only 20 ampères. Account for this by elementary principles. Ind. C. S. 1886.

157. The carbon filament of an incandescent lamp being 15 cm. long and 0·025 cm. in diameter, the current in the lamp 1·5 ampère, the electromotive force between the terminals 50 volts, and the temperature of the filament 2000°, determine the emissive power of its surface for heat. B. Sc. Honours 1884.

ANSWERS

AND HINTS FOR SOLUTION

Chapter I—Dynamics

4. Acceleration = 2·5; space described = 1125 cm. **5.** 20 cm. per sec. **6.** 6666·6 dynes. **7.** 10^5 gm. **8.** 10 min. **9.** 1 hr. 23 min. 20 sec. **10.** 37·5 dynes. **13.** 2·5 ft. per sec. **14.** $v/240$. **16.** 38,400. **17.** (1) 5 oz.; (2) 2 oz. **18.** 32. **19.** 981. **20.** 9,810,000; 1/981. **21.** $4·45 \times 10^5$ dynes. **22.** 785 cm. per sec. per sec. **23.** 12·8; 96,000. **24.** 150 ft. per sec.; 900. **25.** 4 ft. per sec. per sec. **26.** $4·524 \times 10^7$. **27.** 1/120 of a poundal. **28.** The force is to the weight of a gramme as 25,000 to 327. **29.** Acceleration = 1/70; velocity = 1/7. **31.** 8·075 oz. **32.** The force is equal to the weight of 5·6 lbs.; acceleration produced = 0·08 ft.-sec. units. **33.** 1,774,080. **34.** As 80 : 7. **35.** 82·5 sec. **36.** Momentum = 1,471,500. **37.** The force is equal to 3·2 poundals, or is one-tenth of the weight of 1 lb. **38.** As 224 : 675. **39.** $32\sqrt{165}$ ft. per sec. **40.** Acceleration = 4; space described = 50 ft. **41.** Acceleration = 81; tension = 145,800 dynes. **42.** 18 ft. **43.** 3 gm. **44.** 72 ft. **45.** Acceleration $= g/4$; distance $= 2g$. **46.** Tension = 16·8 lbs. weight = 537·6 poundals. **47.** 709·1 cm. per sec. **48.** 5 : 3. **49.** 242·4 ft. **51.** Acceleration = 1 ft.-sec. unit;

space = 6 in. **52.** $g = 950$. **53.** About 9·5 gm. **54.** $g = 980$. **55.** $\sin^{-1}(1/48)$.

57. 26·32 poundals. **58.** Tension = weight of 4024·3 gm. **59.** 0·1113 ft.-sec. units. **62.** $g = 981{\cdot}05$. **63.** $g = 32{\cdot}227$. **64.** It would have to be shortened to 3/19ths of its original length. **66.** $g = 32{\cdot}191$; 39·69 in. **67.** The pendulum is 0·2717 in. too long, therefore 8·15 turns are required to correct it.

77. $9{\cdot}6 \times 10^6$ gramme-centimetres; $9{\cdot}4176 \times 10^9$ ergs. **78.** 288,000 ft.-lbs. **79.** 10 : 27. **80.** $1820\sqrt{3}$ ft.-lbs. **81.** 9600 ft.-lbs. **82.** $1{\cdot}44 \times 10^9$ ergs. **83.** (1) equal; (2) inversely proportional to the masses. **84.** 20·16 ft.-lbs. **85.** 69·12 ft.-lbs. **86.** $6788\frac{4}{7}$ ft.-tons. **87.** In the ratio of 400 to 1. **88.** 1634·3 ft.-tons. **89.** 706·9 ft.-lbs. **90.** Total work = 11,000 ft.-lbs. **91.** 98,175,000 ft.-lbs.

99. (1) 336,000 ft.-lbs.; (2) 10,752,000 ft.-poundals. **100.** 33 ft. **101.** 2500 ft.-lbs. **102.** $3{\cdot}767 \times 10^{10}$ ergs. **103.** $9{\cdot}81 \times 10^7$ ergs. **104.** $6{\cdot}25 \times 10^{10}$ ergs. **105.** $4{\cdot}5 \times 10^{12}$ ergs. **106.** Momentum = $8960\sqrt{2}$; K.E = 143,360 ft.-poundals, or in ft.-lbs. = 143,360/32 = 4480. **107.** 1536 ft.-poundals, or 48 ft.-lbs. **108.** The initial velocity must have been 96 ft. per sec., and the final energy must = K.E. at starting = $5 \times (96)^2/2g = 720$ ft.-lbs. **109.** Force = 300 poundals; K.E. = 225,000 ft.-poundals. **110.** 8×10^{10} ergs. **112.** $2{\cdot}5 \times 10^{10}$ ergs. **113.** 16,940 ft.-lbs. **114.** $8{\cdot}5 \times 10^6$ ergs. **116.** 18 cm. **117.** K.E. = 1210 ft.-tons; a force equal to the weight of $1\frac{5}{8}$ ton. **118.** $5461\frac{1}{3}$ ft.-poundals. **119.** As 112 : 625. **120.** $9{\cdot}375 \times 10^8$ dynes. **121.** 13,090 ft.-lbs. **123.** 32,000 ft.-lbs.; 320 ft. per. sec. **124.** K.E. before impact = 150,000. **125.** Force = 716·8 poundals; work = 114,688 ft.-poundals. **126.** Mass = 45; velocity = 2/3. **127.** As $(\sqrt{3} - 1) : 1$.

130. (1) 4561 kilogramme-metres per min.; (2) $7{\cdot}456 \times 10^9$ ergs per sec. **131.** 38,016 cub. ft. **132.** $9\frac{1}{11}$ H.P. **133.** $52\frac{1}{12}$ miles per hour. **134.** 16·8 H.P. **135.** 151·2 H.P. **136.** 192 H.P. **137.** 480 H.P. **138.** At the rate of 3·367 H.P. **139.** The unit of work would be increased ten-fold; the numerical value of the horse-power would not be changed.

EXAMINATION QUESTIONS

141, 143. See Introduction, § 8, and Ch. I., Exs. 16 and 26. **145, 146.** See §§ 8 and 9 and Exs. 27 and 87. **150.** The proof follows easily when the dimensions $[MLT^{-2}]$ of force are known. **156.** When describing a circle of 100 ft. radius, his inclination to the ice is 84°. **162.** (1) 2000 ft.-lbs.; (2) 1562·5 ft.-lbs. **171.** See Ex. 97. **174.** This example is also inserted (and solved) in the chapter on Thermodynamics (Ch. V., 40). **175.** K.E. of tram-car = 431,500 ft.-poundals; work done in a run of 3 miles = 22,065,120 ft.-poundals.

CHAPTER II—HYDROSTATICS

3. The dimensions of pressure are the same as those of force, viz. MLT^{-2}; the dimensions of intensity of pressure (force per unit area) are $ML^{-1}T^{-2}$. **6.** 124·4 lbs. per cub. ft. **7.** 0·5787 oz. per cub. in. **8.** 1·736 oz. per cub. in. **10.** 13·824. **11.** 10·98 lbs. **12.** 0·7055 gm. per c.c. **13.** 300·8 lbs. **14.** 3·2 cub. ft. **16.** Cross-section = 0·3676 sq. cm.; diameter = 0·683 cm. **17.** 111·97 gm. **18.** 1·4. **19.** 4·5 cm. **20.** As 27 : 10. **21.** 19·712 gm. **23.** 4·516. **24.** 4·57. **25.** $15s/8$. **26.** Its density is 0·925 that of air. **27.** 0·823. **28.** 1·5. **31.** (1) 100 gm. per sq. cm.; (2) 98,100 dynes per sq. cm. **32.** $4{\cdot}273 \times 10^4$. **33.** 73·53 cm. **34.** 4100 gm. weight. **35.** 206,991 dynes per sq.

cm. **36.** 10·193 metres. **37.** 98,100 dynes per sq. cm. **38.** 12,750 gm. weight. **39.** 1/384 lb. per cub. in. **40.** 18,750 lbs. per sq. ft. **41.** 17·36 lbs. per sq. in. **42.** 138·2 ft. **43.** 337,920 lbs. per sq. ft. **44.** 28·02 lbs. per sq. in. **45.** 10·193 metres. **46.** 0·9. **47.** 49·08 ft. **48.** 21·3 lbs. per sq. in. **49.** Sufficient to occupy 5 in. of the tube. **50.** $l\cos 30^\circ(1-s)$. **51.** 150 gm. **52.** 68,360 gm. **53.** 99,000 gm. **54.** 18,150 gm. **55.** 10,000 gm. on upper surface, 11,000 gm. on lower surface, 10,500 gm. on each of the vertical sides.

58. Volume = 20 c.c.; sp. gr. = 3·1. **59.** 22·04 gm. **60.** 21·08 gm. **61.** 34 c.c. **62.** 425·9 oz. **63.** 20·8 gm. **64.** 27·5 gm. **66.** $s_2 = m_2s_1/(m_2s_1 - m_1s_1 + m_1)$. **67.** 20·97 gm. **68.** 21·57 gm. **69.** A weight equal to that of the copper. **70.** Acceleration = $20\frac{4}{7}$ ft.-sec. units; time = 1·247 sec. **71.** 6·04 lbs. **72.** 0·32 gm. **74.** 1000 c.c. **75.** 300 c.c. **76.** 6437·5 cub. yds. **77.** 50 c.c. **78.** Sp. gr. of both solid and liquid = 0·5. **79.** The sphere will rest in equilibrium with 1/7th of its volume immersed in the mercury. **80.** $3\cdot\dot{3}$. **81.** 1·625. **82.** 1·4. **83.** 1·204. **84.** 0·8271. **85.** 1·434. **86.** 0·9127. **88.** 11·31. **89.** 1·948. **90.** 0·7351. **91.** 0·8. **92.** 4·75. **93.** 3·219. **94.** 1·059. **99.** Sp. gr. of pebble = 2·74; of spirit = 0·825. **100.** 240 lbs. **101.** 10,080 lbs. **102.** Ratio of arms = 8 : 5. **103.** Pressure = 8064 lbs.; distance = 0·1042 in. **104.** Mechanical advantage = 1440.

107. Apparent height = $20\sqrt{3}$ in. = 34·64 in. **108.** 34 ft. **109.** 13·6 in. **110.** 25·197 ft. **111.** (1) 0·0938; (2) 1·276. **112.** 11·86 metres. **113.** 10,546 kgm. per sq. metre. **115.** 1,039,890 dynes per sq. cm. **116.** 14·01 lbs. per sq. in. **117.** 30,000 oz. per sq. ft. **118.** 1006·1 gm. per sq. cm., or 10,061 kgm. per sq. metre. **119.** $4\cdot045 \times 10^7$ dynes. **120.** A change of about half a pound (0·4917 lb.) per sq. in. **124.** $p : p' = 9 : 1$.

125. $p : p' = r'^3 : r^3$. **126.** 71·93 cm. **127.** 360 lbs. per sq. in.; 3·6 in. **128.** As 1 : 1·306. **129.** 1·395 gm. **130.** Pressure = 2 atmospheres. **131.** 96·6 cm. **132.** It will descend 1·2 in. **133.** $p : p' = r' : r$. **135.** 54·1 cm. **136.** The tube should have been raised until a length of 80 cm. stood out of the mercury. **138.** 68·85 cm. **139.** About 18 cm. **140.** $\frac{2}{3}$ cub. in. **142.** (1) 34·2 c.c.; (2) 87 c.c. **144.** 4·34 ft. **145.** 4·49 ft. **146.** It must be lowered until its top is 66 ft. below the surface. **148.** 164·3 cub. ft.; 80·3 in. **149.** 0·876 in. **150.** The jar must be sunk until its mouth is 33·64 ft. below the surface. **152.** $D_n = (3/4)^{10}D = 0{\cdot}05631D$. **154.** 0·4363 gm. **155.** 0·741 gm. **158.** 260 kgm.

EXAMINATION QUESTIONS

159. See Ch. VII. p. 172. **164.** See p. 58; pressure = 2379·3 gm. per sq. cm. **166.** Whole pressure = 3750 lbs.; resultant pressure = 625 lbs. **167.** A force equal to the weight of 320 lbs. **172-175.** See pp. 62, 63, and Ex. 171. **189.** See Ex. 137.

CHAPTER III—HEAT

1. **Expansion of Solids.**—**4.** 2·0068 metres; 150°. **5.** 153·86 cm. **6.** 0·0432 in. **7.** 263·16 cm. **8.** 87·2464 cm. **9.** 1·00095 yard. **10.** 22·128 cm. **11.** Coefficient of expansion = 0·00008; temperature = 260°. **12.** 100 cm. **13.** 0·02864 in. **14.** 0·0144 sq. ft. **15.** 0·01764 ft. **17.** 300·912 sq. cm. **18.** 1·00649 metre. **19.** 36 cm. **20.** When the pendulum keeps correct time (*i.e.* at 5°), it swings 86,400 times per day (supposing it to be a seconds pendulum; see p. 30). At 30° its length is increased in the ratio of 1·0003 : 1, and it now swings $86{,}400\sqrt{1/1{\cdot}0003}$ times per day.

Referring to p. 19, it will be seen that $\sqrt{1/1{\cdot}003} = 1/1{\cdot}00015 = 1 - 0{\cdot}00015$ approximately. Therefore the clock loses $86{,}400 \times 0{\cdot}00015 = 12{\cdot}96$ secs. per day. **21.** It will gain 20·52 secs. per day. **23.** The required temperature is 196°·4, and the common length at this temperature is 251·424 cm. **24.** 12°. See § 11 for this and the next example. **25.** Increase in volume = 2·592 cub. in. **27.** 10·22. **28.** 48·373 c.c.

2. **Expansion of Liquids.**—**29.** 0·00002797. **33.** 0·000302. **34.** 0·000301. **37.** 0·0001817. **38.** 103°·8. **45.** 0·0001558. **46.** The coefficient of apparent expansion of the mercury is 0·0001546, and this gives 0·0000274 as the coefficient of expansion of the glass. **47.** 1·57 c.c. **48.** 132°·3 **49.** 109°·65. **50.** 9·517 gm. **54.** The volume of the solid is 12·9752 c.c. at 10°, and $12{\cdot}991\dot{4}$ c.c. at 95°; ∴ its coefficient of expansion is 0·00001468.

3. **Expansion of Gases.**—**58.** 3·187 litres; 54°·6. **59.** (1) 11·16 litres; (2) 12·38 litres. **60.** 221·978 c.c. **61.** 0·10231 gm. **62.** 333°. **63.** 69·63 cm. at 0°; 95·12 cm. at 100°. **64.** 2190·1 c.c. **65.** 0·00367. **66.** 77°·6. **69.** 18°·74. **70.** The temperature must rise from 10° to 57°·16. **71.** 322·6 c.c.

77. As 1 : 0·9269. **78.** 998·9 c.c. **79.** 10·47 atmospheres. **80.** As 1 : 0·7808. **81.** As 1 : 2·256. **82.** The temperature must fall to $-2^\circ\frac{1}{3}$. **84.** 0·599 gm. **85.** 924·9 kgm. **86.** 11·66 litres. **87.** 11·97 gm. **88.** 0·3743 gm. **90.** 1991 c.c. **91.** 37·98 in.; at 459°·5. **92.** As 1 : 1·02. **93.** The temperature must rise to 7°·18. **94.** (1) : (2) = 1 : 0·891. **95.** 299°. **96.** 91°·8. **99.** 13°·65.

EXAMINATION QUESTIONS

100. The lengths must be inversely proportional to the coefficients of expansion. **101.** Observe that the coefficient of *superficial* expansion is approximately double the coefficient of linear expansion; see Introduction, § 11. **103-105.** See footnote, p. 92. **108-113.** See Exs. 51 and 53. **117.** For "50° C." in the last line read "– 50° C."

CHAPTER IV—SPECIFIC AND LATENT HEAT

1. **Specific Heat.**—**5.** 76,800 heat units. **7.** 1995 units. **8.** 4,804,800 units. **9.** 11·875. **11.** 0·208. **12.** Temperature = 17°·14. **13.** 88°. **14.** As 1 : 0·45$\dot{3}$. **15.** $22,666\frac{2}{3}$. **17.** 0·0313. **18.** 0·0962. **19.** 0·615. **20.** 9°·49. **21.** 0·0556. **22.** 0·1327. **23.** $5\frac{5}{9}$ gals. of boiling water, and $14\frac{4}{9}$ gals. of tap-water. **24.** 173°·1. **26.** As 8 : 9. **27.** 68°. **28.** 306·4 units. **29.** 135°·3. **31.** 30 gm. **32.** In the proportion of 1 to 3. **33.** $(s_1t_1+s_2t_2+s_3t_3)/(s_1+s_2+s_3)$. **34.** 0·6426. **35.** 54°·37. **36.** 21·6 gm. **37.** 0·0315. **38.** Exp. I., sp. ht. = 0·03097; Exp. II., sp. ht. = 0·03185; mean value = 0·03141. **40.** Express Q_t and $Q_{t'}$ in terms of a, b, and c: the difference between these $(Q_{t'} - Q_t)$ is equal to the mean sp. ht. $(S_t^{t'})$ multiplied by the difference of temperature $(t' - t)$. Again, let this interval become indefinitely small; in the limit t' coincides with t, and the mean sp. ht. between $t°$ and $t'°$ becomes the true sp. ht. at $t°$ (S_t).

2. **Latent Heat.**—**48.** 6·25 gm. **49.** $10\frac{2}{3}$ lbs. **51.** 176·5 gm. **52.** 80. **53.** $5\frac{5}{14}$ lbs. **54.** 1·125 gm. **55.** 79·5. **56.** 0·0941. **57.** 0·1148. **59.** 15,544·8 pound-degree units. **60.** 11,312 of the same units; 5966·2 lbs. **63.** 1·5 lb. **64.** 0·5625 lb. of ice will be melted, and the result will be a mixture of ice (0·4375 lb.) and water (1·5625 lb.) at 0°. **65.** The temperature will be lowered by 17°·5 (*i.e.* to 7°·5). **66.** 10 lbs. of water at 18°. **67.** The snow will be

melted and raised to 10°. **69.** 3·26 gm. **70.** 3/16 of the water will be frozen. **73.** Contraction = 0·17 c.c. **74.** 888·8 units. **75.** Contraction = 0·1021 c.c. **76.** Contraction = 0·0214 c.c. **77.** Sp. heat = 0·07655. **78.** Sp. gr. of ice $= \frac{11}{12} = 0·91\dot{6}$. **79.** Ice melted = 0·0873 gm.; sp. heat of substance = 0·0814.

87. Vapour-pressure = 6·55 mm.; relative humidity = 0·55 (or 55 per cent). **90.** 31,200. **91.** 21,480 units. **92.** 6·29 lbs. **93.** 536·3 **94.** 37·31 gm. **95.** As 1 : 5. **97.** 541. **98.** 25·04.

EXAMINATION QUESTIONS

100. The amount of heat required to raise a volume v of mercury through $t°$ is $Q = v \times 13·6 \times 0·033t$. To raise an equal volume of alcohol through $t°$ would require an amount $Q' = v \times 0·85st$.

Thus $$\frac{Q'}{Q} = \frac{0·85s}{13·6 \times 0·033} = \frac{s}{0·528},$$

and $\therefore$ $Q' >=< Q$ according as $s >=< 0·528$.

102. The heat will be divided in the ratio of 1 : 12·5, and the rise of temperature will be 1°·85. **104.** Notice that the thermal capacity (8) of the copper vessel must be added to that of the *cold* body (the milk). The sp. heat of the milk is 0·934. **110-112.** See pp. 127, 128. **113.** See Deschanel, *Natural Philosophy*, Part ii. § 435.

CHAPTER V—CONDUCTIVITY AND THERMODYNAMICS

1. Conductivity.—**3.** 5,760,000. **4.** 230,400 units. **5.** 0·16. **6.** $1·476 \times 10^{10}$ units. **7.** 126 kgm. **9.** $0·01\dot{7}$. **10.** 0·0384. **11.** The multiplier for reducing to the C.G.S. system is 0·01.

2. Thermodynamics.—**14.** 7·44 gramme-degrees. **15.** 1390 ft. **16.** $1{\cdot}905 \times 10^{10}$ ergs. **17.** 678 metres. **18.** 0°·7155 warmer. **19.** 4480 gramme-degrees; 194 metres per sec. **20.** 2010·16 watts. **21.** 85,714 gm. **22.** Work done = 3,683,500 ft.-lbs.; rate of working = 24,557 ft.-lbs. per min. **24.** $6{\cdot}77 \times 10^9$ ergs. **25.** 362·46 metres per sec. **26.** The engine is of 67 H.P., and its efficiency is 0·106 (or 10·6 per cent). **29.** 24,396 units. **33.** $2{\cdot}87 \times 10^8$ ergs. **34.** The work done is 2124 ft.-lbs., and the heat-equivalent of this would suffice to raise 1·528 lb. of water through 1° C. **36.** $J = 4{\cdot}2 \times 10^7$.

EXAMINATION QUESTIONS

38. Taking as units the pound, the inch, and the second, the absolute conductivity is 0·00002893. **41.** Efficiency = 3·37 %; heat wasted = 96·63 %. **42.** See Ex. 13. **44.** 1258·7 lbs. **45.** Work done = $1{\cdot}25 \times 10^7$ ergs. **47.** Work done = $2{\cdot}85 \times 10^7$ ergs.

CHAPTER VI—LIGHT

3. 4·41 candle-power. **4.** As 16 : 1. **5.** (1) 1 ft. from candle, between this and the lamp; (2) 2 ft. on other side of candle. **8.** 5 images. **13.** 45°. (Notice that the successive angles of incidence diminish by 15°.) **21.** $p' = 90$ in.; image is 5 in. long. **23.** $f = +15$ cm. (concave). **24.** 8 in. **25.** 30 in. behind the mirror; magnification = 6. **26.** $f = 1$ ft. 1 in.; mirror must be placed 1 ft. from object. **27.** (1) Image is virtual and inverted, 4 in. behind mirror, $\frac{2}{3}$ in. in length; (2) image 8 in. behind mirror, $\frac{1}{3}$ in. in length. **28.** $p = 18$ in., $p' = 36$ in.; image would be half size of object. **30.** The image is real and twice the size of the object; the magnification is the same when $p = f/2$, but in this case the image is virtual. **31.** Distance $= 3f$. **32.** Half that of the object.

38. 8/9; 3/4. **40.** $\sqrt{2}$. **41.** $\sqrt{2}$. **43.** $1{\cdot}\dot{3}$. **45.** 1·525. **46.** $\sqrt{2}$. **47.** Expand the expression for μ given in Ex. 44; it will be found that $\cot \frac{A}{2} = 3{\cdot}566$, and $\therefore A = 31^\circ\ 20'$. **49.** $48'\ 36''$; $1^\circ\ 29'$. **51.** 3°. **55.** 60 cm. **61.** $f = -4\frac{1}{2}$ in. **62.** $p' = -20$ cm.; as 1 : 3. **63.** 10 cm. **64.** 2 ft. 6 in.; equal. **65.** $p' = -2$ ft.; diameter of image $= 1$ in. **67.** 1 ft. from lens, and on the same side as the object. **69.** $f = -37{\cdot}5$ cm.; as 3 : 1. **70.** $p = 1$ ft.; $p' = -3$ ft. **71.** 10 in. from the lens; $f = -8\frac{4}{7}$ in. **72.** $p = 3f$. **73.** (1) $p = 2$ ft.; (2) $p = 1$ ft. **74.** 16 cm. **78.** $F = -10$ cm. **79.** $F = -24$ cm. **80.** $f = +24$ cm. **81.** $f_1 = -3$ in.; $F = -6$ in., and $\therefore f_2 = +6$ in. **84.** 4 cm.; magnifying power $= 20/4 = 5$. **85.** He will be able to see distinctly objects at a distance of 19·2 cm. (*i.e.* his range of distinct vision will be increased by 28·8 cm.) **86.** $f = +20$ cm. **89.** 58·42 cm. **90.** (1) 16·84 cm.; (2) 16·60 cm.; (3) 16·75 cm.: mean $= 16{\cdot}73$ cm.

EXAMINATION QUESTIONS

92. See Ex. 2. The values of x are $+2\frac{2}{3}$ and -24. **96.** $\mu = \sqrt{2}$. **109.** (1) $\lambda = (\sin 2^\circ\ 18')/100 = 0{\cdot}000401$ mm.; (2) $2\lambda = (\sin 4^\circ\ 35')/100 = 0{\cdot}000799$ mm. Mean value of $\lambda = 0{\cdot}0004$ mm. $= 4000$ tenth-metres.

CHAPTER VII—SOUND

1. 341·8 metres per sec. **2.** 30°·4. **3.** 4·39 sec. **5.** 128·1, or almost exactly the note C. **6.** 1261·2 metres per sec. **7.** 129·6 cm. **9.** $2{\cdot}059 \times 10^{10}$. **11.** $2{\cdot}109 \times 10^{12}$. **13.** A fifth (*c* to *g*). **14.** *g″*, *i.e.* a twelfth above the octave of *c*. **15.** 25 lbs.; an additional

11 lbs. (*i.e.* total weight = 36 lbs.) **16.** $1{\cdot}024 \times 10^8$ dynes. **17.** 46·8 vibrations per sec. **18.** Stretching force = $6{\cdot}759 \times 10^6$ dynes = weight of 6·89 kgm. **20.** 440 vibrations per sec. **23.** 460.

EXAMINATION QUESTIONS

24. $v = \sqrt{2} \times 10^5 = 1{\cdot}414 \times 10^5$ cm. per sec. **26.** Vibration frequency = 71. **27.** Length of tube = 1 ft. 2 in. **28.** Frequency = $n \times \sqrt{7/8}$. **29.** 25·6 gm. for 5 segments, and 17·7 gm. for 6 segments.

CHAPTER VIII—MAGNETISM

3. 8 dynes. **4.** 12·8 dynes. **5.** 5. **6.** 24. **8.** 18. **9.** As $\sqrt{2} : \sqrt{3}$. **16.** As 0·417 : 1. **17.** As 0·352 : 1. **18.** 18·93.

EXAMINATION QUESTIONS

25. Torsion in first case is $360° - 30° = 330°$. The required amount of torsion in the second case is 2×330 (for $\sin 90° = 2 \times \sin 30°$). Hence the torsion-head must be turned through $660° + 90° = 750°$.

CHAPTER IX—ELECTROSTATICS

2. -27. **3.** 12 cm. **4.** 8 dynes. **5.** Charge = $\pm 10{\cdot}9$. **8.** Attractive force = 2 dynes; repulsive force = 0·25 dyne. **9.** The resultant force at the third corner is $\sqrt{3}/200$, and acts along the bisector of the angle. **11.** $\sigma = 10/\pi = 3{\cdot}18$. **12.** 198 units. **14.** B will be in equilibrium between A and C at a distance $\overline{AC}/3$ from A; it will also be equally repelled by both when it is placed on CA, produced so that BA = AC.

17. $2\sqrt{2} = 2{\cdot}828$. **18.** Potential is 40; charges are 400 and 600. **19.** Potential 15; charges 1125 and 375. **20.** 5/3 = 16·6. **21.** L; 50. **23.** Distance $= 4\sqrt{5}$ cm.; force = 0·16 dyne. **25.** 8. **27.** As 8 : 1; 52·5 units. **28.** As 2 : 1. **30.** As 9 : 8. **32.** 500.

38. 4284. **39.** 341·8 and 158·2 **40.** Charge on large sphere = 80, on small sphere = 440; common potential = 8. **41.** 34·34 metres.

43. 75,000 ergs. **44.** 300 units; 4500 ergs. **45.** 13,440 ergs. **46.** The potentials are as 3 : 8; the energies of the charges are as 9 : 16. **47.** As 1 : 6. **49.** Potential = 346·4. **51.** As 1 : 3. **52.** As 1 : 5. **55.** The energies of the several discharges (taken in the same order as in Ex. 42) are as 9 : 3 : 4 : 2. **56.** As 4 : 1.

EXAMINATION QUESTIONS

61. The resultant force at the given point acts along the line parallel to the centres of the spheres, and is equal to 0·0025 dyne. **63.** For B 1/4th, and for C 1/16th of the corresponding amount for A.

CHAPTER X.—CURRENT ELECTRICITY

4. 2·823 ohms. **5.** 60 volts. **6.** 50 ohms. **7.** 0·2 ampère. **8.** 5 cells. **9.** Current = $8\frac{1}{3}$ ampères; potential of point B = 80 volts. **10.** Current = 1·2 ampère; potential difference = 1·44 volt. **11.** Original resistance = 15 ohms; additional resistance required = 15 ohms. **12.** 20 cells. **13.** 6 additional cells. **14.** 1·6 ampère. **15.** 4 metres. **16.** The two E.M.F.'s are equal. **17.** Resistance of battery = 60 ohms; resistance of wire $= 28{\cdot}\dot{8}$ ohms. **18.** 1·07 volt E.M.F., and 13 ohms resistance per cell; potential difference = 14·98 volts.

19. E.M.F. of accumulator = 2·2 volts; current after rearrangement = 1 milliampère. **20.** 11 ohms. **21.** Current is increased in ratio of 7 : 10. **23.** 15 cells.

27. 0·6 mm. **28.** $181 \cdot \dot{3}$ metres. **29.** The resistance of B is 81 times that of A. **30.** 72 ohms. **31.** 1·24 ohm. **32.** 6 ohms. **33.** 20·31 ohms. **34.** 0·9434 ohm. **35.** 0·2903 ohm. **36.** As 1 : 4. **37.** 0·007547 ohm. **38.** 872·8 cm. **39.** If r is expressed in C.G.S. units, the specific resistance is $\pi r/40{,}000\,mn^2$. **40.** 0·2188 ohm. **41.** As 5 : 1. **42.** 1080 yards. **43.** 3·04 microhms. **44.** 32·48 microhms. **45.** 0·0783 ohm.

49. 5·411 ampères. **50.** 0·0754 dyne. **52.** 0·206 ohm. **53.** 15 ohms. **54.** 41 ohms. **56.** As 13 : 8.

62. 1·186 gm. **63.** Nearly 2 hours. **64.** 0·05055 ampère. **65.** 0·0003295. **66.** 0·6755 ampère. **67.** 4017 ampères. **68.** 2·658. **69.** 0·3355.

73. The resistance is equal to that of one of the sides. **74.** $3r/4$, where r is the resistance of one of the sides. **75.** 11 ohms. **76.** $C : C' = l's : ls'$. **77.** The current will be doubled. **78.** 89·1 legal ohms. **80.** 4·5 ampères. **81.** The resistance will be diminished by one-fourth. **82.** $6{\cdot}258r$, where r is the resistance of an inch of the base wire. **86.** $C : C' = B(G + S) + GS : (B + G)(G + S)$. **87.** 4 ohms. **88.** 4·5 ohms; 0·5 ampère. **89.** $C : C' = 5 : 6$. **90.** $C : C' = 5 : 3$. **96.** 2 ohms; 2·125 ampères.

100. 2 rows of 2 cells each. **101.** 2 rows of 6 cells each. **102.** 3 rows of 4 cells each. **103.** 4 rows of 12 cells each; current = 0·39 ampère; potential difference = 5·85 volts.

108. $(1) : (2) = (r_1 + r_2)^2 : r_1 r_2$. **109.** As $r_1 r_2 : (r_1 + r_2)^2$ **110.** 0·8 H.-P. **111.** 487·5 watts. **114.** As 3 : 1 in both cases. **115.** 16°·8. **116.** 2·34 watts per candle-power; 32,143 calories. **117.** 0·9165 ampère. **118.** $k = 0{\cdot}00485$.

122. Horse-power = 5·986; efficiency = 85·5 %. **127.** Efficiency = 31·9 %.

EXAMINATION QUESTIONS

131. 0·186 in. diam. **132.** As 3·3 : 2·3. **133.** 0·7643 ohm. **134.** 0·5 ampère; 0·1585 ampère. **136.** $166{\cdot}\dot{6}$ divisions. **137.** 2·163 ohms. **138.** 20 cells. **140.** 1700 ohms. **141.** Deflection of B = 109·9; the reduction-factors of the two galvanometers are as 46 : 25 and their resistances as 25 : 18. **149.** $3{\cdot}6 \times 10^{10}$ ergs. **150.** Specific heat of alcohol = 0·64. **151.** As 1 : 0·0884. **153.** 43,393 gramme-degrees.

	6	7	8	9	1	2	3	4	5	6	7	8	9
10	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
11	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
12	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
13	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
14	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
15	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
16	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
17	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
18	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
19	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
20	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
21	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
22	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
23	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
24	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
25	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
26	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
27	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
28	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
29	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
30	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
31	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
32	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
33	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
34	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
35	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
36	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
37	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
38	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
39	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
40	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
41	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
42	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
43	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
44	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
45	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
46	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
47	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
48	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
49	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
50	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
51	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
52	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
53	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
54	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Table of Natural Sines and Tangents.

Degrees.	Sines.	Tangents.	Degrees.	Sines.	Tangents.	Degrees.	Sines.	Tangents.
1°	·017	·017	31°	·515	·601	61°	·875	1·804
2°	·035	·035	32°	·530	·625	62°	·883	1·881
3°	·052	·052	33°	·545	·649	63°	·891	1·963
4°	·070	·070	34°	·559	·675	64°	·899	2·050
5°	·087	·087	35°	·574	·700	65°	·906	2·145
6°	·105	·105	36°	·588	·727	66°	·914	2·246
7°	·122	·123	37°	·602	·754	67°	·921	2·356
8°	·139	·141	38°	·616	·781	68°	·927	2·475
9°	·156	·158	39°	·629	·810	69°	·934	2·605
10°	·174	·176	40°	·643	·839	70°	·940	2·747
11°	·191	·194	41°	·656	·869	71°	·946	2·904
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13°	·225	·231	43°	·682	·933	73°	·956	3·271
14°	·242	·249	44°	·695	·966	74°	·961	3·487
15°	·259	·268	45°	·707	1·000	75°	·966	3·732
16°	·276	·287	46°	·719	1·036	76°	·970	4·011
17°	·292	·306	47°	·731	1·072	77°	·974	4·331
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